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Original article

Form function for the 'I-214' poplar merchantable stem (Populus × euramericana (Dode) Guinier cv cultivar 'I-214')

Jean-Marc Roda*

AFOCEL, Route de Bonnencontre, 21170 Charrey-sur-Saone, France

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Abstract – This paper describes a research and application integrated procedure: the development, evaluation, and use of a form function for the 'I-214' poplar merchantable stem. Because this form function is to be used by timber merchants, a particular emphasis is placed on its sturdiness and reliability. The model is extrapolated to other poplar clones in order to measure the error when using it beyond the range of validity. The limits of the model and possibilities for its improvement are discussed. Its applications are presented.

stem form / volume determination / taper / equation / broadleaves / simulation

Résumé – Fonction de forme pour la tige marchande du peuplier 'I-214' (Populus × euramericana (Dode) Guinier cv cultivar 'I-214'). Cet article décrit une démarche intégrée de recherche et d'application : le développement, l'évaluation et l'utilisation d'une fonction de forme pour la tige marchande du peuplier 'I-214'. Cette fonction de forme étant destinée à être utilisée de manière concrète par les professionnels de la filière bois, un accent particulier est porté sur sa robustesse et sa fiabilité. Le modèle est extrapolé à d'autres clones de peuplier pour mesurer l'erreur commise lors de son utilisation hors du domaine de validité. Les limites et les possibilités d'amélioration de ce modèle sont discutées. Ses applications sont présentées.

forme de tige / détermination du volume / défilement / équation / feuillu / simulation

1. INTRODUCTION

This paper describes the development and evaluation of a form function for the 'I-214' poplar merchantable stem. This form function must be reliable and easy to use by commercial producers. The function parameters must be correctly predicted in different growth conditions, with limited basic information (total tree height, circumference at 1.30 m).

Poplar is one of the main species of the French forest resource with a timber production of 2.3 millions m^3 in 1996 (second broadleaved species after oak: 2.8 millions m^3). Over the last 37 years, Afocel has established many poplar trials and developed the first French volume table specific to poplar at the national level [5].

The clone 'I-214' is the one for which most data have been collected. It is the major component of plantations that will be harvested in France in the next 10 years. It is still widely planted in some regions.

Very few papers have been published concerning form or taper functions for poplars, except for *Populus tremuloïdes* [6, 11, 15, 17, 20, 21, 22]. Concerning

* Correspondence and reprints
CIRAD-Forêt, TA 10/16, 73 rue Jean-François Breton, 34398 Montpellier Cedex 5, France.
Tel. (33) 4 67 61 44 99; Fax. (33) 4 67 61 57 25; e-mail: jean-marc.roda@cirad.fr

especially the ‘I-214’ clone, Mendiboure [19] has proposed a polynomial form function valid for the department of Isère (France), and Birler [3] has presented equations valid for Turkey (giving ratios for four billets categories). In these two cases, the very restricted application field does not allow practical use in France. Modern calculation and simulation methods allow the creation of better tools than classic volume tables. To build a tool describing the stem form will allow estimation of not only the merchantable volume of standing trees, but also the assortment in terms of billets and particular products with specific characteristics.

2. MATERIALS AND METHODS

2.1. Fitting data

The data come from 23 Afocel trials spread out through 9 departments in the east, north, south, and south west of France (*table I*). These trials are representative of the growth conditions of the ‘I-214’ clone currently planted in France. This good geographical distribution is an essential condition for the reliability of a model expected to be applied at the national level. A total of 2964 trees have been measured. Circumference at 1.30 m ranged from 25 to 165 cm, total height from 7 to 35 meters, and age from 6 to 16 years. Plantation densities ranged from edge alignments to 500 stems ha^{-1} plantation.

In this paper, interest is focused on the merchantable stem, measured to a 7 cm top diameter. The measurement protocol was the following: circumference at 1.30 m, circumferences each meter from 0.5 m to 7.5 m, height to 7 cm top diameter, circumference at half this height, diameter at half the length of the crown log, and total height. That makes 13 circumference or diameter measurements for each tree, or 38532 girth versus height pairs. In addition for each tree, the artificial pruning height and the age are known.

2.2. Extrapolation data

The model was validated on trees taken from a very different population: ‘I-214’ clone on poor soil, and harvested at 25 years (4 plots, 95 trees); ‘Dorskamp’ clone on poor soil with intensive silviculture (1 plot, 19 trees); ‘Beaupré’ clone on good soil with intensive silviculture (4 plots, 140 trees).

Table I. Location and description of trials providing fitting and validation data.

Trial	Dpt[a]	Density (stems ha^{-1})	Site and trial context	Number of measured trees	Measured ages
Cuiserey	21	156	Silt	411	6 to 11 years
Gergy	71	156	Deep silty sand	298	6 to 11 years
Drambon	21	edge	Deep silty sand	204	6 to 12 years
Laissaud	38	204	Deep silty sand	300	7 to 12 years
Pont/vanne	89	159	unknown	69	14 to 16 years
St Marcel	38	208	Thin soil	204	12 to 16 years
La Rochette	38	238	unknown	10	16 years
La Rochette	38	204	unknown	10	16 years
Cruet	38	220	unknown	20	15 years
Détrier	38	500	unknown	20	15 years
Ste Bazeille	47	238	Garonne’s alluvia	113	11 to 13 years
Manses	9	156	Hers’s alluvia	95	12 to 13 years
St Nazaire	38	238	unknown	336	7 to 12 years
St Caprais	31	238	unknown	60	10 to 11 years
St Caprais	31	204	Alluvia	98	9 to 11 years
Bussières	70	196	Insert culture	452	6 to 11 years
Ste Bazeille	47	204	Garonne’s alluvia	107	9 to 11 years
Rambervillers	88	204	unknown	175	8 to 11 years
Le Champ	38	270	unknown	393	5 to 9 years
Drambon Est	21	100	unknown	98	6 to 10 years
Bernin	38	240	unknown	303	6 to 10 years
Sermaize	51	204	Sandy silt, sugar mill wastes	150	6 to 9 years
Frapelle	88	179	Recent alluvia	40	5 years
			TOTAL	3966	

[a] Administrative department.

2.3. Poplar stem form

The stem form characteristic of species with the strong apical dominance typical of conifers [1], is classically represented with a vertical succession of 2 volumes: a truncated neiloid, then a truncated paraboloid (*figure 1*). Plantation poplars although broadleaved, have a high apical dominance, but do not correspond completely to this classical model. The 'I-214' clone form is characterized by 3 superposed volumes [2]. The first one, from the base to first-years branches, is a truncated neiloid. The second one, approximatively the low and medium part of the crown (pruned or not), is a truncated paraboloid. The last one, up to the top, is either a truncated neiloid for trees still dynamically growing, or a truncated cone for mature trees [2, 4]. The detailed graphic study of stem profiles [2] shows that the height at 7 cm top diameter most often corresponds to the junction of second and third volume (*figure 2*). One can say that 'I-214' poplar form corresponds to a volume of a tree of high apical dominance to which is superposed the volume of a well-differentiated top. This phenomenon is without doubt linked to the exceptional poplar growth rate that is tempered in the crown by large major branches, even if this last effect is not as marked as for other broadleaved species such as oak [13].

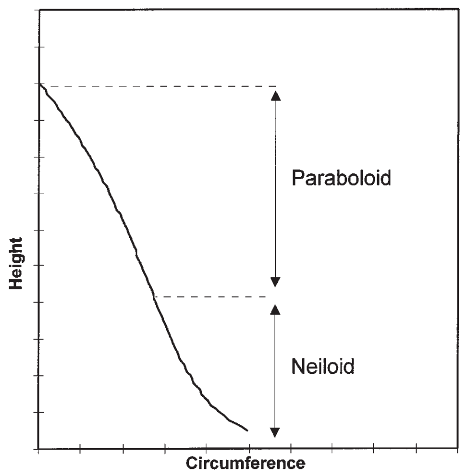

Figure 1. Stem form of species with strong apical dominance.

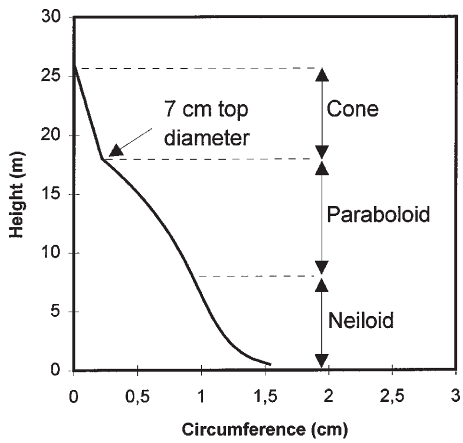

Figure 2. Stem form of poplar, studied by Barneoud et al. [2].

2.4. Model genesis

To enable a possible extrapolation of the model, polynomial form functions with known sturdiness or generality have been tested: Kozak's models and its derivatives [12, 16], Brink's and its derivatives [7, 8, 24, 25], and Pain's [23]. The best results have been obtained with the last four, which are all built on the same principle: the addition of two functions. The first one describes a neiloid for the base of the tree, and the second a paraboloïd for the top of the tree (*figure 3*).

The predictions from these models were, however, not satisfactory for our data. It was necessary to try several supplementary functions derived from these models. One of them has given particularly satisfactory results and was therefore retained for this application. First the parameters were estimated separately for each tree, in order to build a local model. Then relationships between estimated parameters and dendrometric variables for each tree were studied. These relationships allowed the development of a global model for all trees, predicting stem form from simple dendrometric variables.

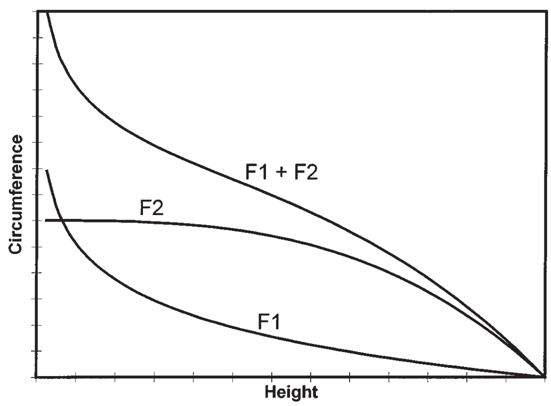

Figure 3. General pattern of models built by addition of functions F1 and F2.

2.5. Local model construction

The Pain's model [23] is:

$$Y = \alpha \cdot (1 + X^3) + \beta \cdot \mathrm{Ln}(X) \quad (1)$$

where Y is the diameter in centimeters, X is the relative height (level above the ground versus total height), α is the parameter characterizing the top of the tree, and β is the parameter characterizing the base curve.

This model has been modified to take account of constraints particular to poplar: the neiloid-paraboloid form characterizes only the merchantable part of the tree, i.e. to 7 cm top diameter [2]. Because there is no commercial interest to model the non-merchantable upper part of the stem, we do not have measurements regarding this part, and we do not need to utilize a segmented equation as described in the literature [9, 10, 18]. Instead of this we consider that a rough linear relationship is enough in order to describe the stem above the 7 cm top diameter, when necessary. Besides, the relative height is replaced by the real height for a direct and easy prediction according to the total height or the height at top diameter.

The modified model gives the circumference in centimeters according to the height in the tree, up to the estimated height at top diameter (*figure 4*):

$$C = 22 + \chi\,[1 - (H/\delta)^3] + \varepsilon \cdot \mathrm{Ln}(H/\delta) \quad (2)$$

where C is the stem circumference in meters, H is the height in the stem in meters, δ is the estimated height at the top diameter, in meters, χ is the parameter characterizing the stem form at half-height, and ε is the parameter characterizing the base of the tree.

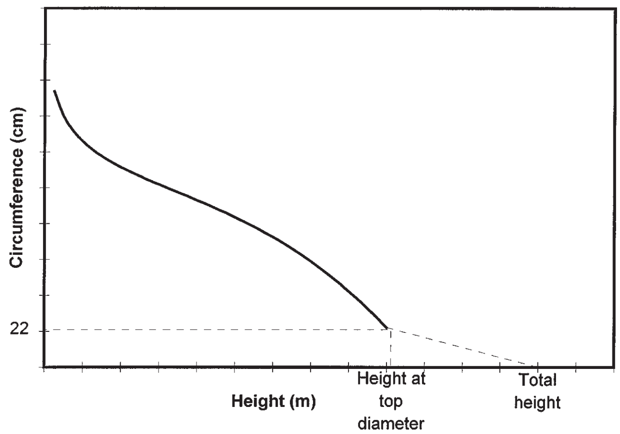

Figure 4. Modified model giving the circumference until the height at 7 cm top diameter.

After a first fitting attempt it was clear that the model was overparameterized, indeed the two parameters δ and χ are correlated and strongly linked to the circumference at 1.30 m. The model has therefore been reparameterized by constraining ε so that the profile passes through the circumference at 1.30 m.

The model is therefore:

$$C = 22 + \chi \cdot [1 - (H/\delta)^3] + \phi \cdot \mathrm{Ln}(H/\delta) \quad (3)$$

where $\phi = [C13 \cdot \delta^3 - (\chi + 22) \cdot \delta^3 + 2.197]/[\delta^3 \cdot \mathrm{Ln}(1.3/\delta)]$ (4)

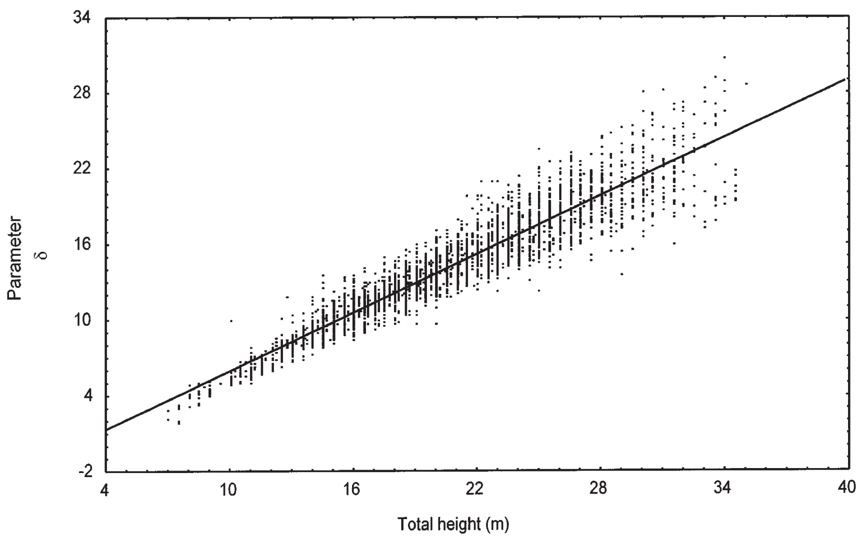

Figure 5. Distribution of parameter δ by tree total height.

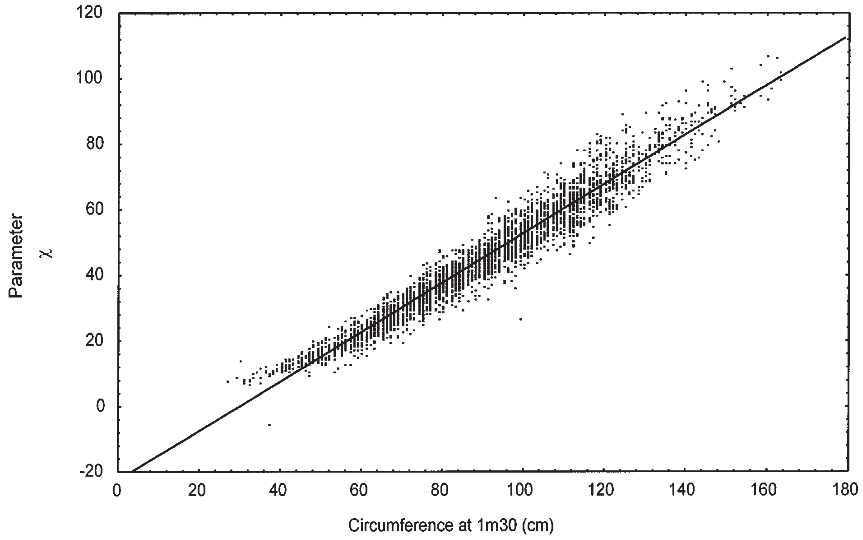

Figure 6. Distribution of parameter χ by circumference at 1.30 m.

with $C13$ being the circumference at 1.30 m.

A second local fitting allowed to study the relationships between parameters and simple, classic dendrometric criteria (circumference at 1.30 m, total height, height to top diameter, density, plot age). Two very strong relationships are apparent: δ was strongly correlated with total height (*figure 5*), and χ was strongly correlated with the circumference at 1.30 m (*figure 6*). Other simple criteria such as the artificial pruning height did not show strong relationships with these two parameters.

Prediction relationships that can be deduced are:

$$\delta = 0.7699 \cdot H_{TOT} - 1.76 \qquad R^2 = 0{,}95 \tag{5}$$

$$\chi = 0.7536 \cdot C13 - 22.575 \qquad R^2 = 0{,}85 \tag{6}$$

where H_{TOT} = total height.

Testing these predictions showed that 64% of trees had less than 5% error on the volume to top diameter prediction, which was judged as satisfactory. However there was a slight bias to the prediction. This bias seems to be due to two major constraints on the merchantable stem form: too great a curvature at the end of the merchantable stem (power equal to 3 in the formulation of the model), and top circumference constrained to 22 cm (i.e. 7 cm diameter). Therefore estimation of these two supplementary parameters was attempted during the development of the global model.

2.6. Global model construction

Replacing local model parameters in equations (3) and (4) by relationships (5) and (6) leads to a global model giving the stem form according to total height and circumference at 1.30 m, with 4 parameters estimated using the 2 964 tree sample. The two supplementary parameters were also estimated using this sample.

In this global model, size is expressed in cross sectional area rather than as circumference. Size is thus closer to stem volume, and gives less weight to errors in the upper part of the merchantable stem during volume calculations.

The model becomes therefore:

For $H < e \cdot H_{TOT} + f$,

$$S = d + \left(a \cdot G13 + b\right)\left[1 - \left[\frac{H}{\left(e \cdot H_{TOT} + f\right)}\right]^{P}\right]$$

$$+ \frac{\left(a \cdot G13 + b\right)\left[\dfrac{1.3}{e \cdot H_{TOT} + f}\right]^{P} + G13 - d - \left(a \cdot G13 + b\right)}{\mathrm{Ln}\left[\dfrac{1.3}{\left(e \cdot H_{TOT} + f\right)}\right]} \mathrm{Ln}\left[\frac{H}{\left(e \cdot H_{TOT} + f\right)}\right] \cdot \tag{7}$$

For $H > e \cdot H_{TOT} + f$,

$S = d/(e \cdot H_{TOT} + f - H_{TOT}) \cdot H + d/[1 - (e \cdot H_{TOT} + f)/H_{TOT}]$ (8)

where S is the cross sectional area at height H, $G13$ is the basal area, H_{TOT} is the total height, H is the level above the ground, a, b, d, e, f, and p are estimated parameters.

3. RESULTS

3.1. Model fitting

Fitting may be assessed using the sum of squared errors (*table II*). The 6-parameter model was retained since the gain on the sum of squared errors was significant in comparison with models where only one supplementary parameter is estimated or even none. The graph of the residuals according to the height allows visualization of whether the fitting is balanced or not, or if any zone distinguishes itself (*figure 7*). In addition, splitting it by trial allows to check whether one can observe this balance in each plot or not (*figure 8*). The residual distribution is less tight for relative heights between 0.35 and 0.65. This zone corresponds to the low part of the crown, between the pruning height and the beginning of the top. Three factors contribute to reducing the precision of the fitting: first, measurements are less precise due to branch insertions; second, there are only very few girth measurements in this part of the stem; third, large branch

Table II. Parameter estimates and summary statistics for the model fitting*.

	Point number	Parameters number	Sum of squared errors (m^4)	Residual error (m^2)	% explained variance	
	32 747	6	0.3692	0.0034	99.09	
Parameters	*a*	*b*	*e*	*f*	*d*	*p*
Estimation	0.6275	–0.0072	0.6701	–0.21143	0.00478	1.9065
Standard error	0.0030	0.0002	0.0026	0.0584	0.00009	0.0160

* The sum of square errors for the 4 parameters model [*a, b, e, f*] is 0.410 m^4; and the sum of square errors for the 5 parameters model [*a, b, e, f, d*] is 0.407 m^4.

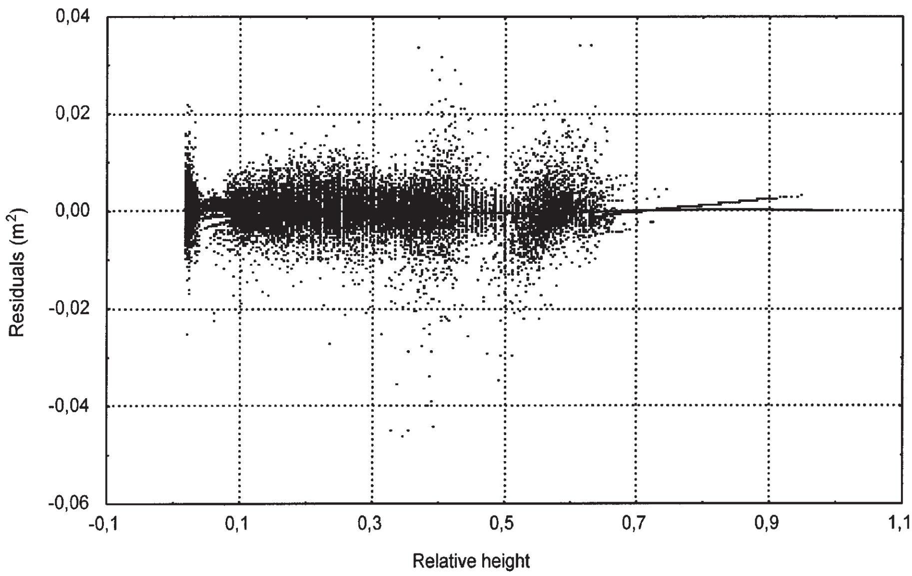

Figure 7. Distribution of cross sectional area residuals on the fitting sample, versus relative height (height of the point in the stem versus tree total height).

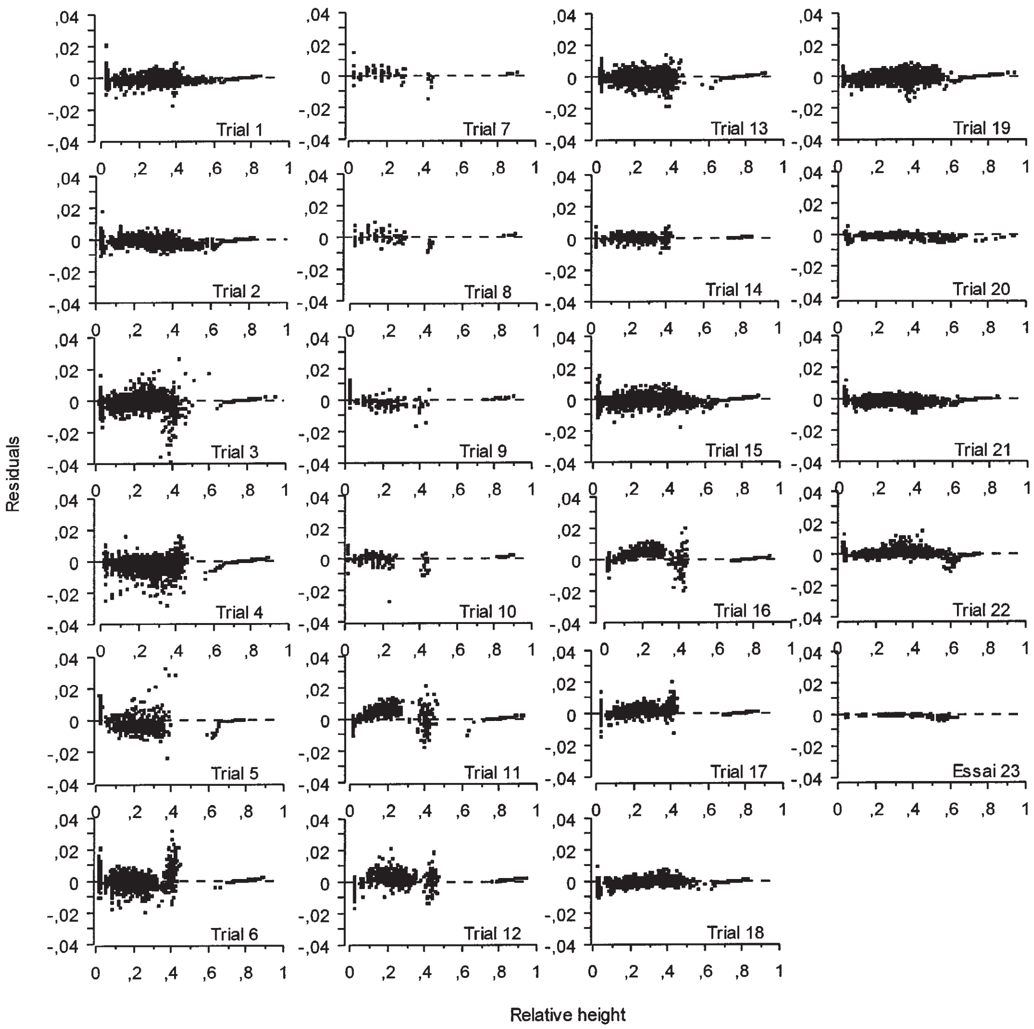

Figure 8. Distribution of cross sectional area residuals on the fitting sample, split by trial.

bases at these heights result in large form variation among individuals.

3.2. Model extrapolation

The model was applied on trees that constitute the extrapolation samples. Predictions from equations determined by the 6 parameters, the cross sectional area at 1.30 m, and the total height of each tree were tested against observed values. Graphs of the residuals about cross sectional area prediction according to the height allow verification of the error distribution (*figure 9*). In these extrapolation samples, relative height to 7 cm top diameter is very variable. But the top diameter height predicted by the model is homogeneous. It results in some dispersion of residuals, diagonally oriented, at the 7 cm top diameter (on either side of the relative height 0.6). However, the cross sectional area is very low in this part of the stem, and prediction errors regarding this part consequently have only a small influence on the volume.

Because of the model's intended use, it was essential to test these predictions at two scales: first at the tree level (volumes of product categories in each stem); then at the plot level (cumulated volumes of product categories in each plot). The main application of this model

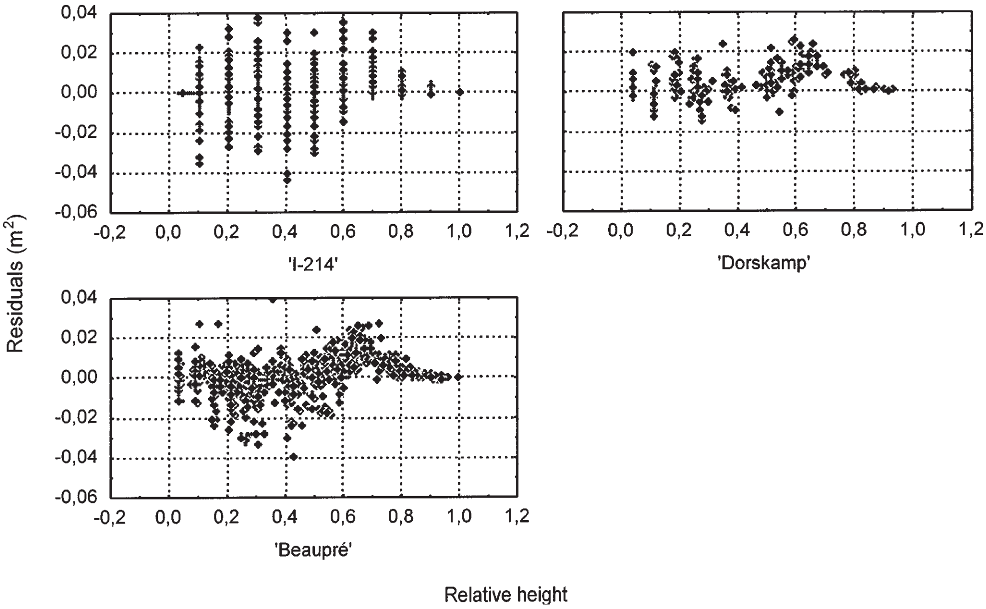

Figure 9. Distribution of cross sectional area residuals predicted on the extrapolation sample.

will be in the assessment of an inventoried parcel, in case of standing sale, or of production forecasting.

Tables III and *IV* present these predictions for three assortment categories (7, 20, and 30 cm top diameters). Compared volumes are observed volumes for each measurement point, and reconstituted volumes after prediction of the cross sectional area at the height of each measurement point. At the plot level, predictions are more precise for the three considered assortment categories. Indeed, errors for each tree tend to cancel out when they are cumulated for the plot. The larger the inventoried plot, the more precise and reliable the prediction.

4. DISCUSSION

The model gives better predictions for a young and intensively cultivated plantation of a different clone than for a 25 years ‘I-214’ plantation. Barneoud et al. [2] and Bonduelle [4] observed a change of the stem form linked

Table III. Predictions of three assortment categories for the extrapolation sample at the tree level.

Tree level	Proportion of trees								
Volume error	7 cm top diameter			20 cm top diameter			30 cm top diameter		
	‘I-214’	‘Dorsk.’[a]	‘Beaup.’[b]	‘I-214’	‘Dorsk.’[a]	‘Beaup.’[b]	‘I-214’	‘Dorsk.’[a]	‘Beaup.’[b]
<5%	32.6%	5.3%	41.7%	49.5%	47.4%	68.4%	72.7%	79.0%	85.0%
<10%	66.3%	68.4%	81.3%	76.3%	94.7%	92.1%	86.3%	100%	95.5%
<15%	87.4%	100%	93.5%	96.7%	100%	96.4%	97.7%	100%	97.7%
<20%	99.0%	100%	98.56%	100%	100%	99.3%	98.9%	100%	100%
<25%	99.0%	100%	100%	100%	100%	100%	100%	100%	100%
<30%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Total number of trees	95	19	139	93	19	139	88	19	133

[a] ‘Dorskamp’ clone; [b] ‘Beaupré’ clone.

Table IV. Predictions of three assortment categories for the extrapolation sample at the plot level.

Plot level	7 cm top diameter			20 cm top diameter			30 cm top diameter		
	'I-214'	'Dorsk.'[a]	'Beaup.'[b]	'I-214'	'Dorsk.'[a]	'Beaup.'[b]	'I-214'	'Dorsk.'[a]	'Beaup.'[b]
Plot volume (m^3)	115.56	30.97	232.34	96.99	28.92	214.33	70.72	22.91	166.81
Prediction error (m^3)	7.16	2.61	2.89	2.91	1.44	–2.38	1.28	0.69	–2.29
Relative error (%)	6.20%	8.43%	1.24%	3.00%	4.98%	–1.11%	1.81%	3.01%	–1.36%

[a] 'Dorskamp' clone; [b] 'Beaupré' clone

to the ageing of poplar plantations. They described the changing of the upper part of the stem from a truncated neiloid for dynamically growing trees, to a truncated cone for maturing trees (or trees growing on poor soil). Because our fitting sample concerns only plantations younger than 16 years, the modelled form is characteristic of young and intensively cultivated poplars. In this case, there is a linear relationship between tree height and the height at 7 cm top diameter. Indeed, values of parameters e and f lead to the relationship (9):

$$\text{“7 cm top diameter height”} \approx 2/3 \cdot H_{TOT} - 0.21. \quad (9)$$

For older trees, there is not such a clear relationship for predicting the height at 7 cm top diameter. The form of the upper part of the stem is probably linked to growth rate, and a better understanding of this would certainly improve the model predictions.

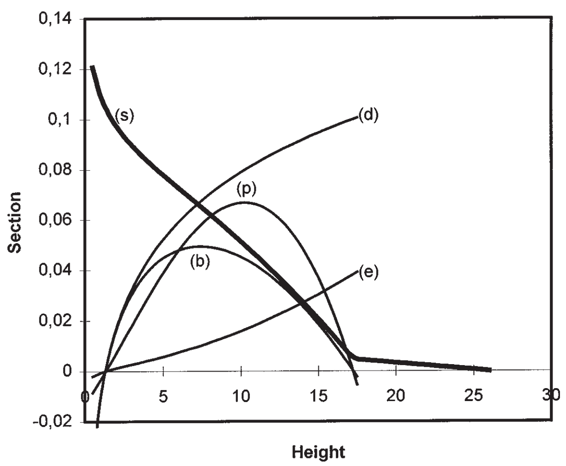

Figure 10. Model sensivity functions are given for a tree of average circumference and height in the modelling sample (height = 26 m, circumference = 115 cm). Corrective factors are applied on these functions in order to trace them at a same scale. (s) is the stem profile (cross sectional area according to the height), (b) is the sensitivity function of parameter *b*, (p) is the sensitivity function of parameter *p*, (d) is the sensitivity function of parameter *d*, and (e) is the sensitivity functions of parameters *e* and *f*.

At the tree level, the prediction of the cross sectional area at the height of the top diameter is not perfect. There is a considerable variability of the observed height at 7 cm top diameter in the fitting sample, and this is even greater in the extrapolation. It seems this variability may be explained by both individual and clonal variation of the large branches in the crown. The configuration of the branch bases is quite different for a clone like the 'Dorskamp', and for this reason a model based on the 'I-214' is less easily applicable in this case. The measurement methodology that has been used for the modelling sample is also implicated, since most circumference measurements in the butt log were below 7.5 m, with only 3 above this height. The model sensitivity functions (*figure 10*) show that it is precisely in this zone above 7.5 m, that it is necessary to get most measurements for the estimation of parameter *d* and of parameters *e* and *f*.

For management purposes, the form equation has been used for the construction of tables giving volumes of standard products. The model data are distributed on the complete range of normal sites for 'I-214', insuring a degree of reliability for these volume tables in France. Moreover the extrapolation has allowed measurement of the committed error for extrapolation beyond the validity field. Constructed tables present different entries so as to be adapted to the different professional practices. Especially, "height" entries that can be selected are either total height or height at a given top diameter.

The equation is the basis of software designed for professionals of the timber sector (*figure 11*). Researched product categories (dimensional criteria: minimal or multiple billet length, given top diameter) are specified by the user. The software calculates the total merchantable volume and volumes for each product, as well as the indicative billet number to be expected in each case. These assessments can be calculated for a given tree, for an average plot, or for an inventoried plot (full or diameter class inventory).

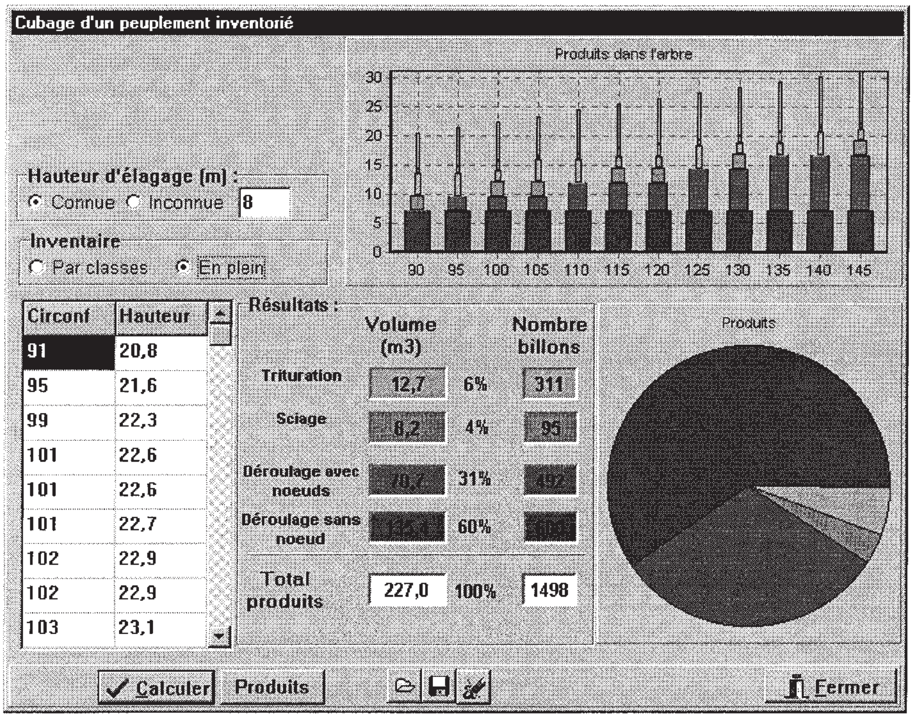

Figure 11. Professional software based on the form function, and developped by Afocel.

5. CONCLUSIONS

The model and derived tools are reliable for intensively managed 'I-214' poplar stands planted on good soils. They also allow good extrapolation for young poplar stands on poor soils or for other clones. However there are some important limits. Two important principles have been brought out for future improvements and widening of the model application field. These future improvements will probably not be seen as a model simplification from the viewpoint of professional practice. In particular, we have seen that prediction of the upper stem form of older trees would have to take growth rate into account. This type of data is rarely accessible to a forest harvester, because he does not always know the plantation age, and because regular measurements are only made in trial and experimental plantations. In the meantime, the model gives adequate results starting only from a knowledge of the circumference at 1.30 m and of the total height of the tree. This constitutes important progress for the culture and the harvesting of poplar stands by providing valuable decision support for professionals of the timber sector, as well as by allowing them to save time and money.

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