*Ann Sci For* (1995) 52, 297-306



**Original article**

# Polyhedral representation of crown shape. A geometric tool for growth modelling

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(Received 14 March 1994; accepted 17 January 1995)

**Summary** — Tree or stand growth modelling often needs an explicit representation of crown shape. This is necessary for crown volume or external surface calculations, or light penetration modelling. Many different representations have been used for this purpose. In this paper, we explore the use of the polyhedral convex hull of the crown as a type of boundary representation. We present an application of this representation for the calculation of geometrical characteristics of common ash trees (*Fraxinus excelsior* L). Crown projection area calculated with the convex hull is closely related to the measured value. Moreover, a strong relation exists between the basal area of the most external tree rings and the convex hull volume or surface area which indirectly validates the polyhedral representations. This relation, however, is no stronger than exists with the simpler crown projection surface area measurement. The convex hull is intermediate in terms of computation costs and efficiency between classical geometric shapes and more elaborate computer graphic representations. It is a simple and versatile tool for modelling purposes.

**crown shape representation / computational geometry / crown volume / convex hull / pipe model / *Fraxinus excelsior* L**

**Résumé — Représentation polyédrique de la forme du houppier. Un outil géométrique pour la modélisation de la croissance.** *La modélisation de la croissance des arbres ou des peuplements fait souvent appel à une représentation géométrique du houppier. Cela est nécessaire pour, par exemple, les calculs de volume et de surface externe du houppier, ou la modélisation de la pénétration de la lumière dans les peuplements. De nombreuses représentations ont été utilisées jusqu'à maintenant, le plus souvent une combinaison de différents solides de révolution. Nous explorons dans cet article les possibilités offertes par la représentation polyédrique convexe où la frontière du houppier est représentée par son enveloppe convexe. Cette représentation est appliquée au calcul des caractéristiques géométriques de houppiers de frênes (*Fraxinus excelsior *L). La projection au sol du houppier, calculée à partir de son enveloppe convexe, est très proche de celle mesurée sur le terrain. De plus,* *une forte relation d'allométrie apparaît entre l'accroissement en surface terrière des cernes les plus externes du tronc et le volume ou la surface de l'enveloppe convexe du houppier. Cette liaison est maximale lorsque l'accroissement est cumulé sur les 3 derniers cernes annuels. Cependant, elle n'est pas meilleure que celle observée avec la surface de projection au sol du houppier, plus facilement mesurable. Ces relations valident indirectement la représentation polyédrique convexe. Cette représentation est un compromis intéressant, en termes de complexité et de précision, entre les solides de révolution classiques et les représentations volumiques plus élaborées.*

***forme du houppier / volume du houppier / enveloppe convexe / géométrie informatique /* Fraxinus excelsior L**

## INTRODUCTION

Crown shape is a key factor in architectural and functional tree modelling. The crown is at the interface between the tree and the atmosphere and as such controls the interception of water, light and pollutants. It interacts directly with other trees by mechanical contact or indirectly by shading. Crown shape both conditions and reflects tree ecophysiological functioning.

Many geometric characteristics of crown shape are used in modelling tree or stand growth. Crown length or horizontal extension are often used for the calculation of competition indices. Crown volume and surface area have been shown to be closely related to foliar biomass (Zeide and Pfeifer, 1991; Jack and Long, 1992; Makela and Albrekton, 1992) or bole increment (Mitchell, 1975; Seymour and Smith, 1987; Sprinz and Burkhart, 1987; Ottorini, 1991). Ecophysiological parameters such as leaf conductance, internal $CO_2$ concentration or water use efficiency are significantly correlated with crown volume (Samuelson *et al*, 1992) and crown surface area has been used for the study of pollution impacts (see *eg*, Dong *et al*, 1989). Geometrical and topological information about the shape of the crown is needed to model mechanical interactions between trees or light interception within stands. Finally, computer graphics also need the use of such data for the synthesis of realistic tree or stand pictures (Reffye *et al*, 1988).

Geometric characteristics are most often obtained from the position of a few distinguishing points of the crown, such as the top, base and maximum horizontal extension of branches. Length (vertical extension) is readily calculated. Horizontal extension is sometimes approximated by the maximum or mean width of the crown, but more often ground crown projection is used. Its area is calculated from the position of intersections between radii centered on the bole and the crown edge projection. Calculation of volume of external surface area of the crown needs reference to an explicit representation of the crown boundary shape. Classical forms used are cylinders, various conics and vertical or radial combinations of these. Koop (1989) presents an extended review of these different forms. His own description of a crown, one of the most elaborate, uses the measured position of 8 points on the crown boundary to fit 4 slices of ellipsoids.

However, these axisymmetrical shapes impose heavy constraints on the representation of the crown boundary. A more relaxed representation can be obtained using a set of points selected on the boundary, and a graph of proximity on this set of points. Various geometric structures can be used for this purpose (Boissonnat, 1984). In this paper, we explore the use of the polyhedral convex hull of the crown, which is one of the simplest structures and has not yet been tested for crown representation. Such a representation can be used as a by-product of architectural models of crown development. These models are based on the quantitative analysis of tree organization at the branch or growth unit level (see *eg*, Mitchell, 1975; Ottorini, 1991; Reffye *et al*, 1991; Prusinkiewicz *et al*, 1993). They imply the precise spatial positioning of phytoelements inside the crown. Thus, they provide the set of data necessary for the polyhedral representation of crown boundary.

We present an application of this polyhedral representation in the calculation of crown shape parameters of common ash (*Fraxinus excelsior* L). To verify the reliability of this representation, we also study the classical allometric relationships between crown dimensions calculated with the polyhedral representation and radial tree growth (see *eg*, Coyea and Margolis, 1992). This work makes use of data initially collected for the modelling of common ash growth development (Cluzeau *et al*, 1994).

## MATERIALS AND METHODS

### Tree sampling and measurements

Twenty-seven common ashes were sampled in various forests of north-eastern France. Trees were chosen in order to represent different ages and crown forms, including free growing trees with a large crown as well as crowded trees with a thinner crown. Before cutting, each tree was measured for diameter at breast height, total height, crown length and crown projection surface area. This latter surface was delimited with a plumbline, from the branches which had the longest horizontal projection all around the tree. Common ash has only a few second order branches, thus 8 to 12 branches were sufficient.

After harvest, annual length increments (growth units) of the branches and the stem were determined. Boundaries between growth units were localized using bud scars. Length, diameter and age of each growth unit were determined. For each main branch, making up the basic crown framework, the azimuth from the north and the length of the leafy part were measured. Second order branches, directly attached to the bole, were distinguished from tertiary branches attached to the secondary branches. From these data, we calculated the Cartesian coordinates (x,y,z) of the origin and tip of each growth unit. A stem analysis gave basal area (at breast height) and bole volume increments for each year. Disks were analysed at 1 year intervals along the bole. For each disk, annual radial increments were measured along 4 radii. A more detailed presentation of sampling and measurements is given in Cluzeau *et al* (1994).

### Calculation of the polyhedral representation and crown shape parameters

All the measured points delimiting each growth unit are included in the crown. From this set of points, we calculated the crown's polyhedral hull. There is no unique solution for this problem, but, among all possible solutions, the convex hull is the simplest and also has some properties that make it easy to manipulate. By definition, the polyhedral convex hull is the smallest convex set containing all the above points. For any pair of points inside a convex set, the segment joining these 2 points is entirely inside the convex set.

The convex hull of the crown was calculated using the gift wrapping algorithm (Preparata and Shamos, 1985) which gives a triangulation of the set of points belonging to the convex hull. Each facet of this convex polyhedron is, by construction, a triangle. We developed an application software for the calculation of convex hulls and image synthesis. This representation allows the calculation of various form parameters.

The position of the center of gravity of this polyhedron was calculated. This gives information on the asymmetry of the crown. Crown length (CL) is the difference between the highest and lowest point ordinates. Total area of the hull (CS) is the sum of the elementary triangular facet areas. Volume (CV) is calculated as the sum of the volumes of elementrary tetrahedra based on each facet and with the centrer of gravity as the summit. Crown projection surface area is the area, on a horizontal plane, of the convex hull of the vertical projections of all the points of the crown convex hull. The surface area of the top part of the convex hull was also calculated since it can be used to estimate the leaf surface exposed to sun. This surface is composed of all the facets which have their normal vector at more than 90° above the horizontal. Finally, the empty interior volume of the crown ("bare inner core", Jack and Long, 1992) and the leafy volume were estimated with the same convex hull approach applied to the set of points delimiting the leafy and leafless zones of the branches inside the crown.

### *Verifying the reliability of the polyhedral representation*

In order to verify the reliability of the polyhedral representation, we compared crown projection surface area calculated with this representation to that measured in the field. Furthermore, allometric relationships between crown volume or surface area and basal area or bole volume increment were studied. Correlation coefficients ($r$) and regression equations were calculated using the SAS package (SAS Institute Inc, 1989). Both raw variables and their squares were tested in the regression equations. The quality of fit was assessed by standard error of the estimates and adjusted coefficient of determination ($R^2$), as well as visual inspection of the residuals. Although consistent with all the calculated regression lines, 1 large tree was removed from the calculations due to extreme and influential values for all variables.

## RESULTS

### *Calculation and graphical representation of the polyhedral representation*

Figure 1 shows 1 tree and figure 2 its polyhedral convex hull. For each figure, the tree is represented from 2 different directions, shifted by 5°. This allows the reconstruction of a 3-D view of the tree using a classical stereoscope. The complexity of the convex hull increases with the number of branches from 16 facets for the smallest tree to 66 facets for the biggest, corresponding to 10 and 35 points, respectively (table I). The empty interior volume is small compared to the total volume (10% on average). Calculated values of volume are rather low in comparison with values observed elsewhere. Vrestiak (1989) observed average values of 800 $m^3$ for free growing common ashes at 50 years old.

### *Allometric relationships between crown dimensions and growth*

Figure 3 gives the relationship between the measured and calculated crown projection surface area. The correlation is very high ($r = 0.98$), indicating that our representation gives a valid view of the real crown, in 2-D space at least. A slight underestimation occurs for the largest trees, above 25 $m^2$ of the crown projection surface area.

Table II gives the linear correlation coefficients between various crown shape parameters calculated with our convex representation (surface area and volume of the external convex hull, crown projection surface area) and measurements of tree growth (annual basal area and bole volume increments). A strong allometric relation exists between the measured tree basal area and calculated crown surface area ($r = 0.82$) or volume ($r = 0.81$). However, the correlation is even better with measured crown projection surface area ($r = 0.89$).

Interestingly, the correlation of surface area, volume of the convex hull, or the crown projection surface area, with squared basal area increment cumulated from the most external ring over the last 10 years, reaches a maximum for the 3 external rings ($r = 0.93$ with the convex hull surface area, $r = 0.92$ with its volume, and $r = 0.93$ with measured crown projection surface area). It is interesting to note that this relationship (fig 4) holds for all trees in our sample, either free growing or suppressed. An analysis of the residuals of this regression shows that over-

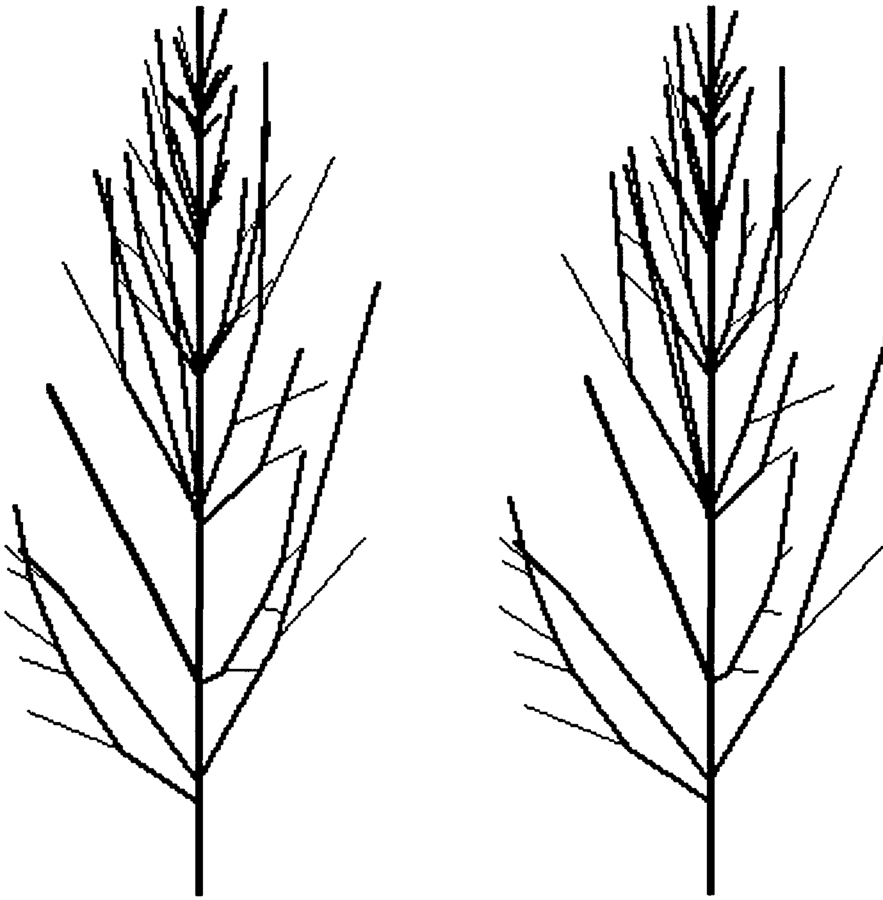

**Fig 1.** 3-D representation of a common ash tree. Only shoots to the 3rd order are represented. The right and left images are views of the same tree 5° horizontally shifted. This allows the reconstruction of a 3-D view using a classical stereoscope (with a 7 cm distance between the glasses).

estimation of the crown volume occurs when very low branches are developed down the main stem. In this occurrence of outliers, convexity assumes the presence of a continuous layer of leaves from these low branches upwards, whereas these branches are isolated at the base of the tree, without any leaves.

The equations of the regression lines are:

$$CS = 18 + 174\ BAI^2 \quad R^2 = 0.85 \quad n = 26$$
$$CV = -0.6 + 164\ BAI^2 \quad R^2 = 0.83 \quad n = 26$$

where CS is the convex hull surface area ($m^2$), CV is the convex hull volume ($m^3$), and BAI is the squared surface increments of the 3 most external rings at breast height ($dm^2$).

These 2 relations present an efficient way to rapidly calculate crown volume and surface area based on simple measurements of external tree-ring increments.

Neither the top part of the crown surface nor the leafy volume were better correlated with radial growth than total crown surface

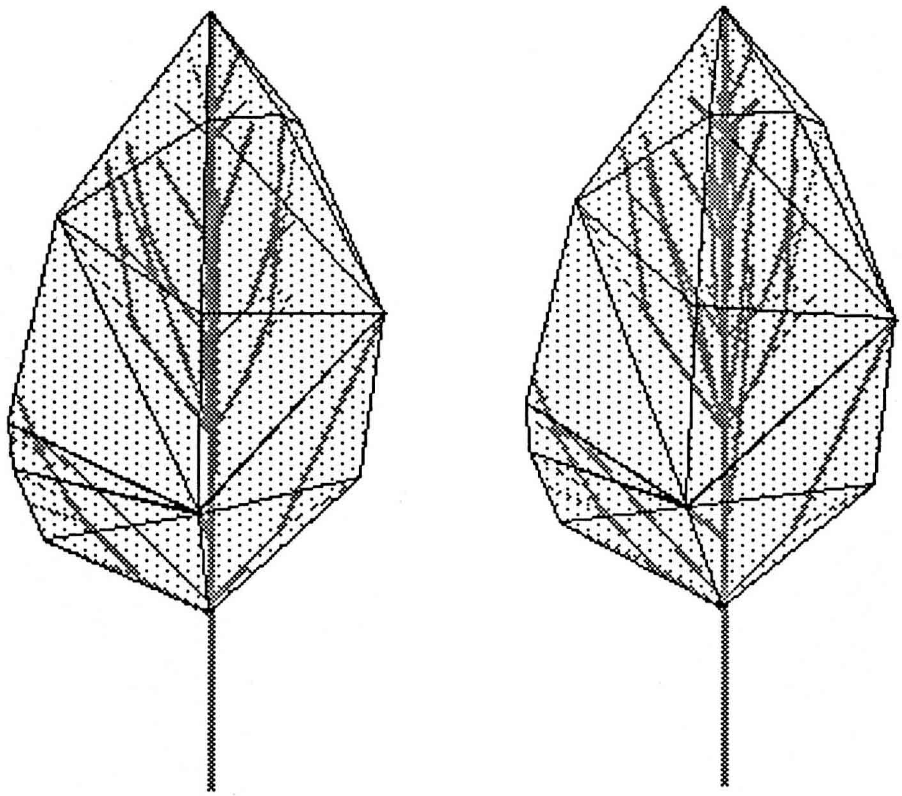

**Fig 2.** Convex hull of the crown of the previous common ash tree. The tree is viewed from a 45° angle above the horizontal plane. The crown projection surface is also shown. Other comments as in figure 1.

and volume (table II). These relations were not improved by removing 3rd order branches from the convex hull calculation.

## DISCUSSION

Crown projection surface area is the only measurement we have to directly verify the validity of our representation. The agreement between measured and calculated values is very high. However, this is only a 2-D validity test of our 3-D representation. Validity of the polyhedral representation is also strengthened by the very strong correlation we observed between current increments at breast height and both crown surface area and volume. This relationship between conducting and evaporating surfaces (the so-called pipe model) has been extensively documented (see Coyea and Margolis, 1992). It is a consequence of the fact that hydraulic conductance in stems is the product of conducting tissue area multiplied by the specific conductivity of these tissues. For ash trees, the relationship is better with a squared value of conducting surface area. This is in accordance with the theoretical hydraulic conductance, given by the Hagen-Poiseuille equation, which is proportional to the 4th power of the capillary radius (Ewers and Zimmermann, 1984).

**Table I.** Medians, minima and maxima for measured tree attributes and crown dimensions calculated from the polyhedral representation ($n = 27$).

| *Measured tree attributes* | *Median* | *Minimum* | *Maximum* |
|---|---|---|---|
| Age | 23 | 12 | 62 |
| Basal area ($cm^2$) | 176 | 20 | 616 |
| Total height (m) | 14.8 | 7.0 | 22.6 |
| Crown projection surface area ($m^2$) | 9.0 | 1.1 | 55.5 |
| *Convex hull calculations* | | | |
| No of points lying on the convex hull | 16 | 10 | 35 |
| No of facets delimiting the convex hull | 28 | 16 | 66 |
| Crown surface area ($m^2$) | 45.4 | 9.2 | 195.4 |
| Top surface area of crown ($m^2$) | 21.4 | 2.5 | 125.5 |
| Crown volume ($m^3$) | 19.6 | 1.3 | 206.4 |
| Empty interior volume ($m^3$) | 2.0 | 0.1 | 34.7 |
| Leafy volume ($m^3$) | 18.1 | 1.1 | 196.8 |
| Crown projection surface area ($m^2$) | 6.4 | 0.7 | 42.0 |

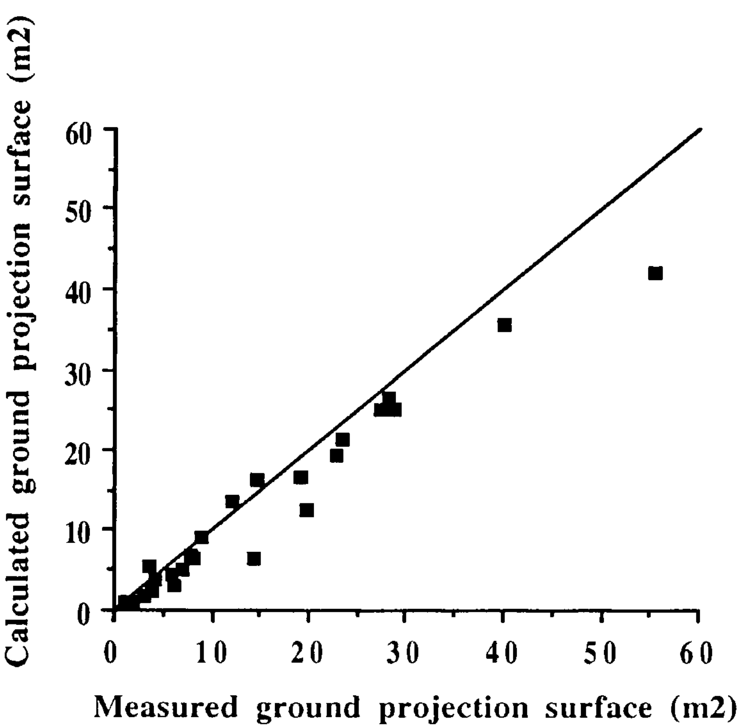


**Fig 3.** Relationship between calculated and measured crown projection surface area ($r = 0.98$; $n = 26$). The equality line is added.

**Table II.** Correlation coefficients between convex estimators of crown shape and tree growth measurements ($n = 26$). All correlations are significant at the 1‰ level.

| | *Measured growth parameters* | | | |
|---|---|---|---|---|
| | *Crown projection surface area* | *Tree basal area* | *Squared basal area increment at bh* | |
| | | | *Of the last ring* | *Of the last 3 rings* |
| Measured crown projection surface area | 1 | 0.89 | 0.78 | 0.93 |
| *Calculated convex hull parameters:* | | | | |
| Crown surface area | | | | |
| Total | 0.96 | 0.82 | 0.85 | 0.93 |
| Top part | 0.88 | 0.65 | 0.79 | 0.91 |
| Crown volume | | | | |
| Total | 0.96 | 0.81 | 0.81 | 0.92 |
| Leafy part | 0.96 | 0.79 | 0.83 | 0.93 |

The correlation of both crown volume and surface area is highest with the area of the last 3 annual rings. This suggests that water flow in common ash could be restricted to these external rings rather than stretching throughout the sapwood. A similar observation was made for oaks (Rogers and Hinckley, 1979) and Norway spruce (Sellin, 1993), and direct measurements confirmed the assumption for oaks (Granier *et al*, 1994). Hence, the functional value of the morphologically defined sapwood as an

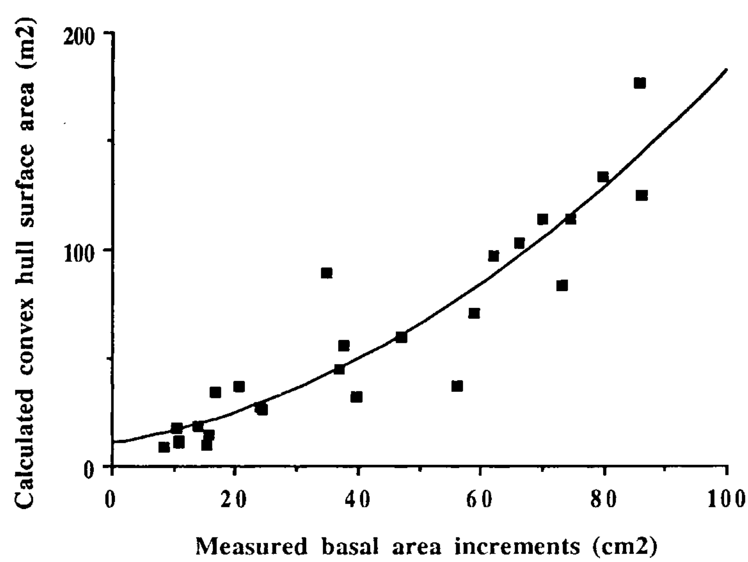


**Fig 4.** Relationship between external surface area calculated as the area of the convex hull, and measured basal area increment during the last 3 years ($R^2 = 0.85$; $n = 26$) for the regression on squared values of basal area increments). The quadratic regression line is added.

indicator of xylem conductive surface is suspect and the necessity of direct measurements of water flow area within the trunk is made apparent.

However, the calculated crown external surface area is not a better predictor of radial growth then the measured crown projection surface area. This means that, for a growth model of common ash using an allometric relationship between radial increment of bole and crown structure, the simple measurement of the crown projection surface area gives a sufficient estimation. On the contrary, Maguire and Hann (1988) observed a better correlation of sapwood area with crown surface area then with simpler variables.

The polyhedral representation is theoretically more precise than that of the axisymmetrical solids because the real crown shape is often randomly built by contacts with neighbors, shading or illumination and various injuries (insect attacks, snow and ice damage, *etc*). Developmental asymmetry can be very important. Thus, the polyhedral representation is closer to the real shape. Whereas no overestimation of crown surface area, volume nor crown projection surface area was apparent in our results, the polyhedral representation is sensitive to extreme outliers. This could be improved by calculating non-convex hulls, thus allowing the representation to account for depressions on the crown boundary (Boissonnat, 1984). Finally, convexity seems to be an acceptable constraint on the representation of crown shape since concave shapes are seldom observed in nature, especially for broadleaves (Koop, 1989).

The intrisic limit of the polyhedral representation is that it is only a boundary representation of the crown. Thus, the internal distribution of leaves inside the crown is assumed homogeneous. Rather than homogeneous, this distribution could be fractal (Zeide and Pfeifer, 1991). The architectural arrangement of foliage in the tree crowns has been shown to strongly control radiation absorption, photosynthesis and transpiration (Wang and Jarvis, 1990; Whitehead *et al,* 1990). Non-boundary representations, such as the computer technique of voxel space, can handle the internal heterogeneity of distribution of phytoelements (Green, 1989).

However, the need for such more elaborated representations of the crown depends on the study scale and final objectives. Polyhedral representation is very efficient in terms of computation time and memory space requirements for computer graphics and calculations of light penetration at the stand level. It is less demanding than voxel representations. Computation requirements and accuracy of representation for the different methods are roughly opposed and correlated with the number of points used for the crown representation, from 8 points in the Koop's model (1989), to 10–35 points for our polyhedral representation, to hundreds of points in voxel spaces. Therefore, the polyhedral representation offers an interesting alternative to these solutions. Due to the large number of data needed for its calculation, it is best suited when the position of growth units within the crown is already known, either from previous measurements or as a by-product of an architectural model of crown development.

## ACKNOWLEDGMENTS

The authors are indebted to JM Ottorini and N Le Goff who were involved in the initial steps of this work. We also thank R Canta and L Garros for technical assistance during sampling and measurements.

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