Simulation of wood deformation processes in drying and other types of environmental loading*

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(Received 3 October 1994; accepted 19 October 1995)

Summary – Deformation processes in wood exposed to drying and other types of environmental loading are simulated by use of the finite element method. In the material model applied, the orthotropic structure of the wood material is considered. The differences of properties in the longitudinal, radial and tangential directions for stiffness parameters as well as for moisture shrinkage parameters are taken into account. As an illustration of possible application areas, the deformation development of boards during drying is simulated. In the analyses, the influence of spiral grain and the variation of wood properties with the distance from the pith are considered. The simulation yields information about unfavourable deformations that develop during the drying process.

simulation / deformation / wood / moisture / finite element method

INTRODUCTION

The moisture content of a growing tree is high, and it is normally necessary to dry the timber before using it for construction purposes. During industrial drying of wood, it is important to avoid excessive deformation of the sawn timber. The deformation process is affected by variations of the moisture and temperature conditions. To mi-
nimize unfavourable deformations, such as cup, twist, crook and bow (see fig 1), one may optimize the environmental conditions during the drying process. To do this, it is helpful to perform numerical simulations of the deformation process.

Characteristic of wood is that its behaviour is strongly orthotropic due to the internal structure of the material and very dependent on moisture and temperature. In addition, the material is characterized by a strong variation of the properties in the radial direction. Another important property which affects the behaviour of wood is spiral grain, causing the direction of the fibres to deviate from the longitudinal direction of the tree. Furthermore, the behaviour of wood is strongly affected by variations in the environmental conditions, especially when the material is exposed to stress.

Simulations of deformation processes are very complex and require a suitable numerical method. In the present work the finite element method is applied.

MODELLING OF MATERIAL PROPERTIES

Theoretical simulation of the deformation process of wood during drying or other types of moisture variation requires a proper constitutive model. The orthotropic structure of the material has to be considered, and it is also important to consider the fact that the behaviour of wood is strongly influenced by variations in the environmental conditions.

In the constitutive model used in the present work, the total strain rate $\dot{\varepsilon}$ is simply assumed to be the sum of the elastic strain rate $\dot{\varepsilon}_e$, moisture strain rate $\dot{\varepsilon}_m$, and mechano-sorptive strain rate $\dot{\varepsilon}_n$, i.e.,

$$\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_m + \dot{\varepsilon}_n \quad [1]$$

This means that creep and possible crack development are not taken into account in the present paper. In the following, the strain rate components will be expressed and a relation between stresses and strains will be given.

Elastic strain

The elastic strain is related to the stress by Hooke’s law, i.e,

$$\varepsilon_e = C\sigma \quad [2]$$

where $C$ is the compliance matrix and $\varepsilon_e$ and $\sigma$ are the elastic strain and stress, respectively.

Denoting the longitudinal, radial and tangential directions by $l$, $r$ and $t$, respectively, the matrices $\varepsilon_e$, $\sigma$ and $C$ are given by (see e.g., Bodig and Jayne, 1982):

$$\varepsilon_e = \begin{bmatrix} \varepsilon_l & \varepsilon_r & \varepsilon_t \ y_l & y_r & y_t \ \gamma_l & \gamma_r & \gamma_t \ \gamma_{lr} & \gamma_{rt} & \gamma_{tt} \end{bmatrix}^T \quad [3]$$

$$\sigma = \begin{bmatrix} \sigma_l & \sigma_r & \sigma_t & \tau_{lr} & \tau_{rt} & \tau_{tt} \end{bmatrix}^T \quad [4]$$

Fig 1. Deformation types.
The parameters $E_l$, $E_r$, and $E_t$ are moduli of elasticity, $G_{lr}$, $G_{rl}$, and $G_{tr}$ are shear moduli and $v_{lr}$, $v_{rl}$, $v_{lt}$, $v_{tl}$, $v_{tr}$, and $v_{rl}$ are Poisson's ratios.

**Moisture induced strain rate**

The moisture induced strain rate is assumed to be dependent on the rate of change of the moisture content only, and is defined as

$$\varepsilon_{\omega} = \alpha \dot{\omega}$$  \[6\]

where $\dot{\omega}$ denotes the rate of change of moisture content and $\alpha$ is defined as

$$\alpha = [\alpha_l \alpha_r \alpha_t 0 0 0]^T$$  \[7\]

The parameters $\alpha_l$, $\alpha_r$, and $\alpha_t$ are material coefficients of moisture induced strain. Above the fibre saturation point $w_f$, these coefficients are assumed to be zero.

**Mechanosorptive strain rate**

If a wood specimen under load is allowed to dry, it exhibits greater deformation than the sum of the deformation of a loaded specimen under constant humidity conditions and the deformation of a nonloaded drying specimen. This phenomenon is called the *mechanosorptive effect* and is in the present work assumed to be given by a generalization of the expression suggested by Ranta-Maunus (1990).

$$\varepsilon_{\omega_o} = m \sigma \lvert \dot{\omega} \rvert$$  \[8\]

This generalization has been described by Santaoja (1990), Thelandersson and Morén (1990) and Santaoja et al (1991). In Eq [8], $\lvert \dot{\omega} \rvert$ denotes the absolute value of the rate of change of the moisture content and $\sigma$ is the stress. The matrix $m$ is a mechanosorption matrix which is defined as

$$m = \begin{bmatrix}
    m_l & -\mu_{lr} & -\mu_{rt} & 0 & 0 & 0 \\
    -\mu_{lr} & m_r & -\mu_{rt} & 0 & 0 & 0 \\
    -\mu_{rt} & -\mu_{rt} & m_t & 0 & 0 & 0 \\
    0 & 0 & 0 & m_{rl} & 0 & 0 \\
    0 & 0 & 0 & 0 & m_{tr} & 0 \\
    0 & 0 & 0 & 0 & 0 & m_{rt}
\end{bmatrix}$$  \[9\]

where $m_l$, $m_r$, $m_t$, $m_{rl}$, $m_{rt}$, $m_{tr}$, $\mu_{lr}$, $\mu_{rl}$, $\mu_{rt}$, $\mu_{tr}$ and $\mu_{rt}$ are mechanosorption coefficients.

**Stress-strain relation**

Eqs [1] and [2] can be combined to form

$$\sigma = D \varepsilon - \sigma_o$$  \[10\]

where the matrix $D$ is the inverse of the compliance matrix $C$ in Eq [2] and $\sigma_o$ is a so-called pseudo-stress vector which describes the effect of moisture change and is given by

$$\sigma_o = D (\varepsilon_{\omega_o} + \varepsilon_{\omega_o})$$  \[11\]

The stress-strain relation given by Eq [10] has been expressed in a local system of coordinates, with the axes parallel to the longitudinal, radial and tangential directions (the orthotropic directions). To perform a simulation of a board, this stress-strain relation has to be transformed with respect to a global system of coordinates, in order to consider the fact that the orthotropic directions vary with the position in the board studied.

**FINITE ELEMENT FORMULATION**

A finite element formulation for simulation of deformations and stresses in wood during drying is given by

$$K\ddot{u} = \dot{P} + \dot{P}_0$$  \[12\]
where \( \dot{a} \) is the rate of nodal displacement vector and \( K, \dot{P} \) and \( \dot{P}_0 \) are stiffness matrix, load vector and pseudo-load vector, respectively, given by

\[
K = [B^T \bar{D} B] dV \tag{13}
\]

\[
\dot{P} = \int N^T t dS + \int N^T f dV \tag{14}
\]

\[
\dot{P}_0 = [B^T \bar{\sigma}_0] dV \tag{15}
\]

and where \( N \) and \( B \) are shape functions and strain shape functions for the element type used, and \( t \) and \( f \) are surface load and body force, respectively. In the present work, small strain analysis is applied and \( B \) is therefore not affected by the displacements. Due to the fact that the orientation of the material varies with the position in the board, the matrices \( \bar{D} \) and \( \bar{\sigma}_0 \) have to be computed using transformation matrices which are specific to each material point considered. This means that \( \bar{D} \) and \( \bar{\sigma}_0 \) are related to \( D \) and \( \sigma_0 \) of Eq [10] by the relations

\[
\bar{D} = G^T DG \tag{16}
\]

\[
\bar{\sigma}_0 = G^T \sigma_0 \tag{17}
\]

in which, eg, \( a_{lx} \), is the cosine of the angle between the local \( l \)-direction and the global \( x \)-direction. In a case where the \( l \)-direction coincides with the \( x \)-direction and \( \theta \) is the angle between the \( r \)-direction and the \( y \)-direction, the matrix \( G \) can be written

\[
G = \begin{bmatrix}
    a_{lx} a_{ly} a_{lz} & a_{ly} a_{lx} a_{lz} & a_{lz} a_{lx} a_{ly} & a_{lx} a_{ly} a_{lz} \\
    a_{lx} a_{ly} a_{lz} & a_{ly} a_{lx} a_{lz} & a_{lz} a_{lx} a_{ly} & a_{lx} a_{ly} a_{lz} \\
    2a_{lx} a_{ly} & 2a_{ly} a_{lx} & 2a_{lz} a_{lx} + a_{ly} a_{yz} & 2a_{lx} a_{ly} + 2a_{lz} a_{yz} + a_{ly} a_{lz} \\
    2a_{lx} a_{ly} & 2a_{ly} a_{lx} & 2a_{lz} a_{lx} + a_{ly} a_{yz} & 2a_{lx} a_{ly} + 2a_{lz} a_{yz} + a_{ly} a_{lz}
\end{bmatrix} \tag{18}
\]

The displacements and stresses are computed by solving Eq [12] using a time-stepping procedure. The theory of the finite element method will not be further described here, but it can be studied elsewhere (see eg, Ottosen and Peterson, 1992 or Zienkiewicz and Taylor, 1989 and 1991).

**MATERIAL DATA**

For simulations of moisture induced deformations, a relevant description of material parameters in the longitudinal direction is important. In a study by Wormuth (1993), the distribution of the elastic modulus in the longitudinal direction has been investigated for Norway spruce (\textit{Picea abies}). Boards cut into specimens with a cross section of 9 x 9 mm were studied. The distribution of the elastic modulus in the longitudinal direction for one board is illustrated in figure 2. The highest value of the elastic modulus is about twice as large as the lowest value.

In figure 3, the values of figure 2, together with the values of another board, are shown as a function of the distance from the pith. It can be observed that the distance from
the pith has a very strong influence on the elastic modulus in the longitudinal direction. The relation between distance from pith and longitudinal elastic modulus may with good agreement be represented as $E_l = 9.7 \cdot 10^3 + 1.0 \cdot 10^5 \frac{r}{r_p}$ Mpa, with $r_p = 1.0$ m, which is also shown in figure 3.

The specimens used by Wormuth (1993) were used by the authors of the present paper to determine the longitudinal moisture elongation coefficient $\alpha_l$. Also for this parameter, a very strong dependence on the distance from the pith has been observed. In figure 4, the distribution of $\alpha_l$ for the same board as in figure 2 is shown.

The relation between the distance from pith and the longitudinal moisture elongation coefficient $\alpha_l$ for the boards of figure 3 is illustrated in figure 5. The coefficient $\alpha_l$ is assumed to be related to the distance from the pith $r$ by $\alpha_l = 7.1 \cdot 10^{-3} - 3.8 \cdot 10^{-2} \frac{r}{r_p}$, with $r_p = 1.0$ m, which is also shown in the figure.

According to experimental evidence (see eg. Mishiro and Booker, 1988), the direction of the fibres deviates from the longitudinal direction of the tree. The deformation of wood during drying is to a large extent dependent on the direction of the fibres. In the present simulation, the spiral grain angle is assumed to be $\phi = 3 - 13.6 \frac{r}{r_p}^2$, with $r_p = 1.0$ m.
THREE-DIMENSIONAL SIMULATION OF BOARD DEFORMATION

To gain information about the shape stability of kiln-dried timber it is helpful to simulate the cup, twist, crook and bow deformation caused by a change of moisture content. This section presents results from a simulation which has been performed using a commercial finite element program (Hibbitt et al, 1993) and a mesh with 6 x 12 x 40 eight-node solid elements with 2 x 2 x 2 integration points. Since mechanosorptive strain according to Eq [8] was not available in the standard version of this program, elastic and moisture induced strains only were considered. This seems to be a reasonable approximation in this case as the stresses are expected to be relatively small. The material was assumed to dry from a moisture content of 0.20 to 0.10. Four boards were studied with a cross section of 50 x 100 mm, a length of 3 m and different orientations in the log and material parameters, as shown in figure 6.

No external constraint was assumed. Displacements were prescribed to avoid rigid body motions only. The deformation

\[ E_I = 9.7 \cdot 10^3 + 1.0 \cdot 10^5 \, r/r_r \, MPa \]
\[ E_T = 500 \, MPa \]
\[ G_{II} = 300 \, MPa \]
\[ G_{III} = 800 \, MPa \]
\[ G_{IV} = 500 \, MPa \]
\[ G_{V} = 50 \, MPa \]
\[ \nu_{II} = 0.40 \]
\[ \nu_{III} = 0.40 \]
\[ \nu_{IV} = 0.50 \]
\[ \alpha_I = 7.1 \cdot 10^{-3} - 3.8 \cdot 10^{-2} \, r/r_r \]
\[ \alpha_r = 0.19 \]
\[ \alpha_l = 0.35 \]
\[ \phi = 3 - 13.6 \, r/r_r \, ^\circ \]
\[ r_r = 1.0 \, m \]

**Table 1.** Deformation of boards according to simulation.

<table>
<thead>
<tr>
<th>Board</th>
<th>Cup (mm)</th>
<th>Twist (°)</th>
<th>Crook (mm)</th>
<th>Bow (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.63</td>
<td>7.75</td>
<td>0.00</td>
<td>1.01</td>
</tr>
<tr>
<td>S2</td>
<td>0.38</td>
<td>1.22</td>
<td>0.00</td>
<td>3.83</td>
</tr>
<tr>
<td>S3</td>
<td>0.22</td>
<td>0.59</td>
<td>2.27</td>
<td>3.10</td>
</tr>
<tr>
<td>S4</td>
<td>0.00</td>
<td>0.43</td>
<td>4.59</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Fig 6.** Orientation of boards studied and material parameters used.
obtained in the simulation is illustrated in figure 7. In table I, the cup, twist, crook and bow, evaluated as defined in figure 8, for the four boards are listed. It should, however, be noted that, in the present analysis, elastic and moisture dependent strain, only, are taken into account, and consideration of the mechanosorptive strains would probably affect the results. Nevertheless, the results show that the deformation development is strongly dependent on the way the board has been cut from the log. It can be observed that the board close to the pith has the strongest twist deformation, due to the spiral grain. This result has been experimentally confirmed by Perstorper (1994).

TWO-DIMENSIONAL SIMULATION OF A KILN-DRYING PROCESS

It is of great value to obtain information about the deformation occurring during kiln-drying of wood. In this example, this application has been chosen to illustrate the capabilities of simulation of deformation development. When interest is focused on studying the deformation parallel to a cross section of a board, a two-dimensional simulation may be performed. In the present application it was assumed that the same conditions are valid for any cross section along the longitudinal axis of the board.

Since, in a board drying without constraint, the stresses $\sigma_l$ as well as the strains $\varepsilon_l$ in the longitudinal direction are in general not zero, the state is neither plane stress nor plane strain. The material model previously described includes coupling between stresses in the longitudinal direction and

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Fig 7. Deformation of boards according to simulation.

Fig 8. Definition of deformation modes used in evaluation.
Fig 9. Cross section studied and material parameters used.

\[ E_t = 400 + 2200 \ (w_f - w) \ \text{MPa} \]
\[ E_i = 220 + 1300 \ (w_i - w) \ \text{MPa} \]
\[ G_{11} = 25 + 72 \ (w_f - w) \ \text{MPa} \]
\[ \nu_{rt} = 0.55 \]
\[ \alpha_t = 0.19 \]
\[ \alpha_f = 0.35 \]
\[ m_r = 0.15 \ \text{MPa}^{-1} \]
\[ m_i = 0.20 \ \text{MPa}^{-1} \]
\[ m_{11} = 0.80 \ \text{MPa}^{-1} \]
\[ \mu_{rt} = 1.0 \]

Fig 10. Development of cross section deformation (left) and deformation due to to cut of a surface lamella (right).
strains in the transversal directions. If, however, this coupling is neglected, only the stress components $\sigma_r$, $\sigma_t$, and $\tau_{rt}$ have to be included in the analysis and a two-dimensional simulation can be performed in a straightforward manner. The simulation has been performed using the program CAMFEM (Dahlblom and Peterson, 1982) and a mesh with $10 \times 30$ plane four-node elements, each built up of four triangular subelements of constant strain type. The cross section of the board studied and the material data used are shown in figure 9. The board was not subjected to any external constraint. Displacements were prescribed to avoid rigid body motions only.

The present simulation was focused on the modelling of deformation development and the moisture transport was assumed to be governed by a linear diffusion relation. To get a realistic time scale for the drying, the diffusivity was chosen as $D_w = 7 \cdot 10^{-10}$ m$^2$/s, the density as $\rho = 400$ kg/m$^3$, the initial uniform moisture content 0.30 and the surface moisture content 0.10, which yields approximate agreement with experimentally observed variation of moisture content, obtained by Samuelsson (personal communication).
communication). The description of moisture distribution applied qualitatively reflects the conditions in a drying board. It should, however, be noted that, in a detailed simulation, the nonlinearity and direction dependence of moisture transport in wood has to be considered (see eg, Claesson and Arvidsson, 1992; Perré et al., 1993; Ranta-Maunus, 1994). Computed deformation of the cross section at four different times during the drying process is illustrated in figure 10 (left). The cupping after 6 days of drying is predicted to be about 1.4 mm. Due to the fact that shrinkage in the tangential direction is greater than in the radial direction, a great cupping deformation is developed. To gain information about the internal stress distribution of a drying board, a surface lamella may be cut. When the lamella is cut from the board, the constraint of the lamella will be released, and deformation occurs. The magnitude of the deformation depends on the stress in the lamella. This type of test has been simulated by disconnecting elements at the position of the cut at four different times, as shown in figure 10 (right). The results shown in figure 10 resemble the results obtained experimentally by Samuelsson (personal communication; see fig 11).

CONCLUSION

The present paper describes numerical simulation of deformation in wood during drying and other environmental loading. Finite element simulations give valuable information on the importance of different material properties for the development of unfavourable deformation. It may be concluded that the variation of material parameters with respect to the distance from the pith must be considered and that spiral grain is an important parameter for prediction of deformation development in wood exposed to moisture variation.

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