Ann Sci For (1996) 53, 885-898

Original article

Prevision of the bending strength of timber with a multivariate statistical approach

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(Received 1 December 1994; accepted 17 November 1995)

Summary – This paper describes a multivariate analysis applied on maritime pine lumber properties, leading to a statistical modeling of the bending strength of this material using indirect nondestructive information. Correlation models are used in this study to estimate the distributional characteristics of concomitent properties that must be specified in European strength classes of lumber (prEN338), mainly density, modulus of elasticity (MOE) and modulus of rupture (MOR). The basic steps of research are described as follows: i) Develop the statistical multivariate model, using the empirical distributions and correlation matrix obtained from a database. ii) Simulate the distributions of MOE and MOR with incomplete information on density, using Monte Carlo simulations to generate random variables. iii) Grade the simulated population according to prEN standards, with one nondestructive estimator (NDE) and estimate the characteristics of each class. Comparison between simulated and experimental histograms of MOE and MOR indicates that the prediction of the first moments of distributions (mean and standard deviation) is better than the prediction of the lower tails. Simulations of grading using density or MOE as NDE have been made for different populations of beams. It is found that a slight increase in average density increases significantly the percentage of simulated high grade lumber (C_{30}). Finally, the comparison between simulated results and a visual grading according to the NFB 52-001 standard clearly shows the efficiency of statistical models for grading and design purposes.

timber / NDE (nondestructive estimation) / strength / correlation models / maritime pine

Résumé – Prévision de la résistance du bois en flexion par analyse statistique multivariable. *Cet article décrit une modélisation statistique multivariable appliquée aux propriétés mécaniques en flexion du bois de pin maritime en dimension d'emploi, dont le but est la prévision de la résistance en flexion à partir d'une information non destructive. Les modèles de corrélation sont utilisés ici pour évaluer les distributions de grandeurs requises dans la classification européenne (prEN338) : densité, module d'élasticité en flexion (MOE), contrainte de rupture (MOR). Les principales étapes de la recherche sont : i) le développement d'un modèle multivariable, à partir des distributions empiriques des variables et de la matrice des corrélations, ii) la simulation par la méthode de Monte Carlo du MOE et du MOR en utilisant comme information la moyenne et l'écart type de densité, iii) le classement des populations simulées selon les spécifications de la prEN338, à partir d'un estimateur non destructif (END). La comparaison des distributions simulées et expérimentales montre une bonne prévision des deux premiers moments statistiques (moyenne et écart type), les valeurs caractéristiques du MOE et* *du MOR étant en revanche sous estimées. Des simulations de classement mécanique ont été effectuées sur trois populations de bois à faible, moyenne et forte densité, montrant une augmentation significative des pourcentages de pièces classées. Les pourcentages simulés se rapprochent d'un classement optimum fait sur la base de donnée ; en revanche le même classement effectué sur des critères visuels (NFB 52-001) sous-estime les caractéristiques effectives des bois.*

bois de charpente / MOR (module de rupture) / prévision / modèles de corrélation / pin maritime

INTRODUCTION

Recent developments in the prediction of the mechanical properties of wood members have focused on nondestructive estimation and correlation models, with special interest in lumber and glulam products (Pellicane, 1984, 1993; Hernandez et al, 1992; Rosowsky, 1994). To this end, important experimental programs have been realized in the past decades in order to assess the distributional characteristics of concomitent usual properties for various wood species. This has been done for instance in France at CTBA (Centre technique du bois et de l'ameublement) for most important French-grown conifers. Multivariate statistical analysis provides useful information concerning the determinism of wood behavior (Castéra and Morlier, 1994) and can be usefully applied in grading and design procedures. These aspects are actually taken into account in the application of European standards (prEN338) which define limiting conditions for a population of lumber entering a given strength class C_{xx} on at least three correlated variables: density (D_{12}), longitudinal modulus of elasticity in bending (MOE) and bending modulus of rupture (MOR) (Rouger et al, 1993):

<u>Strength class</u> C_{xx}

Modulus of rupture	$MOR_{0.05} > \mathbf{XX}$,
Density	$D_{12\,0.05} > \mathbf{D_{lim}}$,
Modulus of elasticity	$MOE_{mea} > \mathbf{MOE_{lim}}$

The conditions for MOR and D_{12} correspond to the 5% fractile (characteristic value) of the respective frequency distribution functions, which means that the actual expected values have a probability of $P = 0.95$ greater than the required conditions. For MOE, the limit holds on the average value. Among these three parameters one at least, the MOR, cannot be directly assessed.

Multivariate models are used in this study to calculate, through Monte Carlo simulations, the joint distributions of D_{12}, MOE and MOR for given strength classes, using a statistical information on one or several nondestructive estimators (NDE): visual, physical or mechanical parameters. The main objective of the research is to compare the efficiency of various NDE in predicting the quality of lumber according to European standards. In a first step, a single input variable, D_{12}, is used to simulate the characteristics of lumber. Density usually exhibits significant correlations with the mechanical properties of wood. However, the significance level of correlation is affected by the type of lumber (juvenile or mature wood, proportion of defects), and the density-based model could therefore be improved by additional information concerning the sample composition. For instance, the juvenile wood effect on the distributional characteristics of MOE and MOR in fast grown species was recently discussed by Tang and Pearson (1992), and it was shown by the authors that juvenile wood affected significantly the elastic properties of lumber.

Visual grading methods, based on a visual assessment of quality, and stress grading methods, based on a direct measurement of MOE, are then introduced in the multivariate modeling. In the first case, the influence of knots on the bending strength is taken into account through the knot area ratio (KAR), which is usually calculated over the whole cross-section area (KAR_{tot}), or in the tension zone only (KAR_{ten}). Because it is often assumed that failure will occur at the most critical defect, the maximum KAR value along the beam, and its distributional characteristics, are used in simulation procedures.

The multivariate statistical analysis was performed on a database composed of 56 maritime pine trees of similar dimensions coming from fast growing stands (young trees with a large proportion of juvenile wood), and traditionally managed stands (older trees). Beams were collected at different positions along the stem. The influence of sample composition on the characteristics of the model are analyzed in the second part of the paper. The correlation model is then used to simulate and compare various grading procedures.

APPLICATION OF A MULTINORMAL MODEL TO NON-NORMAL VARIABLES

Background

Predicting the distributions of dependent properties has been the subject of many papers in recent years (Pellicane, 1984; Taylor and Bender, 1988, 1991; Richburg and Bender, 1992). The general form of a multivariate model can be written as follows:

$$P(X_1) = P(X_1 = x_1)$$

$$P(X_1 / X_2) = P(X_2 = x_2 / X_1 = x_1)$$

$$P(X_3 / X_1 \text{ and } X_2) = P(X_3 = x_3 / X_1 = x_1 \text{ and } X_2 = x_2) \quad [1]$$

$$\ldots$$

$$P(X_i / X_1, X_2, \ldots, X_j) = P(X_i = x_i / X_1 = x_1, \ldots, X_j = x_j)$$

in which $P(X_i/X_j)$ are conditional probabilities and X_i, X_j dependent variables. In particular, when X_i and X_j are random normal variables, equation [1] leads to the multinormal model, which is of common use in reliability. Application of this model requires the mean vector $\{\overline{X}_i\}$ and the covariance matrix COV (X_i,X_j) of the set of variables. In many engineering problems, however, the normal distribution does not fit well the experimental histograms, which are often bounded and dissymetric. The most common probability density functions (pdf) used in such cases are the lognormal and the Weibull pdf. The Weibull law is an extreme value distribution and is often used to represent the variations of strength properties in brittle materials. The lognormal pdf can be applied to variables for which a lower bound needs to be defined. One advantage of the lognormal distribution is that the multinormal model can be used in a logarithmic space of the variables. A multivariate model was also proposed by Pellicane (1984) using S_B pdf to fit the marginal distributions of variables.

Simulation of correlated data

As indicated previously, the multinormal model can apply on non-normal dependent vectors after transformation into a standard normal space of variables (Der Kiureghian and Liu, 1986). The procedure used to generate concomitent non-normal variables with a multinormal model has been described by Taylor and Bender (1991), and can be summarized as follows: i) Generate a vector {N} by Monte Carlo simulations, containing standard random normal observations. The dimension of {N} is equal to the number of variables. The terms of {N} are independent. ii) Apply the correlation matrix to {N}. This procedure will generate a vector {Z} of correlated observations which follow a normal distribution. iii) Evaluate $\Phi(Z)$ $(0 \leq \Phi(Z) \leq 1)$ for each observation where Φ is the cumulative normal distribution function. iv) Apply the inverse of the original marginal cumulative distribution of each variable. The result will be a vector {X}, containing observations of random variables ($X_1,...X_n$) which have the same marginal distributions and covariance matrix as the non-normal studied variables. This processus can be summarized by:

$$\{Z\} = [L]\{N\} \quad [R] = [L][L]^t \qquad [2a]$$

$$X_i = F^{-1}[\Phi(Z_i)] \qquad [2b]$$

where [L] is the Choleski decomposition of the correlation matrix [R].

The matrix [L] for the case of three variables is:

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ \rho_{12} & \sqrt{1-\rho_{12}^2} & 0 \\ \rho_{13} & \rho_{23/1}\sqrt{1-\rho_{13}^2} & \sqrt{1-\rho_{13}^2}\sqrt{1-\rho_{23/1}^2} \end{bmatrix}$$

where ρ_{ij} is the coefficient of correlation between X_i and X_j, and $\rho_{ij/1}$ is a partial correlation coefficient:

$$\rho_{ij/l} = \frac{\rho_{ij} - \rho_{il}\,\rho_{jl}}{\sqrt{1-\rho_{il}^2}\ \sqrt{1-\rho_{jl}^2}}$$

Since the correlation is carried out on a normal space, the coefficients of correlation should be calculated on the normal vector {Z}. When applied on non-normal variables, a correction factor, F, must be applied on the coefficients to obtain the 'normalized' matrix L. This factor has been calculated for various combinations of probability distributions by Der Kiureghian and Liu (1986). Generally, F is a function of the coefficient of correlation ρ_{ij} and the coefficients of variation COV_{Xi} and COV_{Xj}, and the form of this function depends on the choice of pdf of X_i and X_j. Rosowsky (1994) approximated an expression of the correction factor using a polynomial function of $\rho_{MOE,\ MOR}$, COV_{MOE} and COV_{MOR}, when MOE has a lognormal pdf and MOR a Weibull pdf. The numerical expression of F proposed by Rosowsky is then:

$$\begin{aligned} F = {} & 1.031 + 0.052\rho_{MOE,MOR} + 0.011COV_{MOE} \\ & - 0.21COV_{MOR} + 0.002\rho_{MOE,MOR} \\ & + 0.35COV^2_{MOE,MOR} + 0.005\rho_{MOEMOR}\,COV_{MOE} \\ & - 0.174\rho_{MOE,MOR}\,COV_{MOR} + 0.009COV_{MOE}COV_{MOR} \end{aligned}$$

Another method for evaluating the 'normalized' matrix [L] is to calculate directly the correlation matrix on the 'normalized' data Z_i given by the following transformation on the experimental values X_i

$$Z_i = \Phi^{-1}[F(X_i)]$$

Equation [2] has been used in the following analysis to predict the variability of strength properties with different NDE.

STATISTICAL ANALYSIS AND SIMULATION OF DATA

Database

The multivariate approach has been applied on a database of maritime pine (*Pinus pinaster* Aiton) timber properties. The sample used to fit the distributions and estimate the correlation matrix was composed of 615 full size beams (40*100*2 000 mm^3). The population of trees from which beams were collected could be divided into five age classes corresponding to different growth rates. One aspect of the research was the estimation of growth rate influence, in terms of harvesting age, on lumber quality of maritime pine (Castéra et al, 1992); the different classes are defined as follows: age 1 (30- to 40-year-old trees), age 2 (40- to 50-year-old trees), age 3 (50- to 60-year-old trees), age 4 (60- to 70-year-old trees) and age 5 (70- to 80-year-old trees).

The statistical analysis was carried out on D_{12}, MOE, MOR and KAR_{tot}. The dimensions and weight of each beam were used to estimate an average dry air density, and a correction factor was applied to account for the actual moisture content of the beam, which could differ slightly from one specimen to another.

Two point loading bending tests were used for MOE and MOR measurements. The distance between loading points (corresponding to the pure bending zone) equals one-third of the span, and the maximum KAR_{tot} value between loading points was recorded. The MOE is derived from the slope of the moment/curvature curve in the central part of the beam, obtained from ramp loading tests with a constant rate of displacement. A characteristic curve is illustrated in figure 1. The slope is calculated in the linear range of the curve (approximately 30% of the ultimate strength). The bending strength is derived from the load at failure (the type of failure was usually brittle). All beams that failed outside of the central zone were excluded from the analysis.

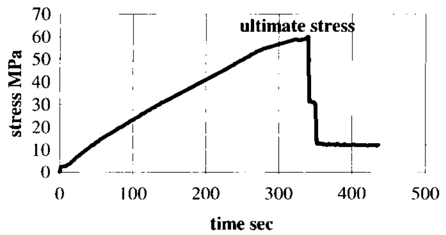


Fig 1. Typical stress versus time curve obtained from a ramp loading bending test up to failure. MOE is calculated from three ramp loading tests in the linear range of the curve. Failure is usually of brittle type.

Fitting of data

The statistical parameters of experimental distributions are presented in table I for each age class and each variable. The level of uncertainty on KAR_{tot} is very important compared to other variables (the COV of this variable is around 75%), regardless of the age of the trees. Besides this, a significant proportion of beams contained no defect in the central part, leading to minimal values of KAR_{tot} equal to 0, as indicated in table I. The average value of KAR_{tot} is not significantly affected by age, whereas D_{12} and the mechanical properties increase with age. The increase of the proportion of juvenile wood in lumber from fast growing trees is one possible interpretation of this result.

The distributional characteristics estimated from goodness-of-fit analysis on the whole sample are given in table II. Choice of the theoretical pdf was governed by Kolmogoroff-Smirnoff statistics performed on the complete distribution. A normal distribution was chosen for D_{12}, whereas the lognormal and 3P-Weibull pdf gave the best results for MOE and MOR, respectively. KAR_{tot} follows specific patterns which cannot be correctly fitted by common pdf. In further simulations we shall assume this parameter to be normally distributed around an average value.

Correlations

Table IIIa and b gives the initial and corrected (normalized) coefficients of correlation for the whole database. Examples of correlations are shown in figure 2. No significant differences appear between the initial and the normalized coefficients.

As expected, the best NDE of strength is the MOE. On the other hand, MOE is also significantly correlated to D_{12}, and the partial coefficients $\rho_{MOR,\ KAR/MOE} = -0.43$ and $\rho_{MOR,\ D12/MOE} = +0.11$ indicate that one can expect a better prediction of strength using MOE and KAR_{tot} than MOE and D_{12}. The comparison between the different regression equations is shown in table IV.

The relationship between KAR and the bending strength probably could be improved by considering the position of knots with respect to the tension side of the beam, especially for small KAR values for which the residual variability on strength is quite large.

Simulation of MOR with different correlation models

Five models were tested: three of them (A, C, E) used D_{12} as initial NDE, model B was only based on the MOE/MOR correlation,

Table I. Descriptive statistics of timber characteristics according to the age of trees.

	Age class	Sample size	Mean	SD	Minimum	Maximum	1st quartile	3rd quartile
Density (kg/m^3)	1	80	535	43	452	644	502	562
	2	98	527	46	439	614	487	565
	3	105	552	45	462	672	516	583
	4	103	575	51	488	741	541	607
	5	111	574	47	485	718	537	604
	All sample	498	554	50	439	741	516	586
MOE (GPa)	1	id	9.05	1.67	5.05	14.19	7.76	10.33
	2	id	9.50	2.10	4.91	14.28	7.71	11.23
	3	id	10.18	2.24	5.76	16.11	8.26	11.75
	4	id	11.30	2.30	6.67	16.77	9.38	13.10
	5	id	11.13	2.13	6.86	16.93	9.58	12.39
	All sample	id	10.30	2.30	4.90	16.90	8.50	11.90
MOR (MPa)	1	id	41	19	12	93	25	53
	2	id	42	21	13	85	24	60
	3	id	46	22	14	100	26	62
	4	id	51	22	13	100	30	70
	5	id	52	22	19	106	33	70
	All sample	id	46	22	12	106	28	63
KAR_{tot} (%)	1		29	20	0	97	11	43
	2		23	19	0	70	3	35
	3		25	16	0	59	14	38
	4		20	18	0	57	1	34
	5		23	18	0	97	7	33
	All sample	414	24	18	0	98	7.5	37

( )* is the coefficient of variation. See text for the description of classes and the measurement of properties. MOE: modulus of elasticity in bending; MOR: modulus of rupture; KAR_{tot}: knot area ratio calculated over the whole cross section area.

Table II. Probability density functions (pdf) of the timber characteristics obtained from goodness-of-fit analysis.

Timber	*Normal (m, σ)*	*Lognormal 2P (μ, s)*	*Weibull 3P (x_0, k, ω)*
KAR_{tot} (%)	(24, 18)	–	–
MOE (GPa)	–	(2.37, 0.248)	–
MOR (MPa)	–	–	(11.2, 1.61, 39.3)
Density (kg/m^3)	(554, 50)	–	–

See table I for abbreviations.

and models D and E combined KAR_{tot} with MOE and D_{12}, respectively, to predict strength: A: Density/MOR, B: MOE/MOR, C: Density/MOE/MOR, D: KAR/MOE/MOR, E: Density/KAR/MOE/MOR.

Before comparing the models, it is necessary to study the accuracy of numerical simulations for each model, and determine the numerical error on simulated parameters. Plots in figure 3 compare the experimental and simulated coefficients of correlation between MOE and MOR obtained by simulations with 100, 500 and 1 000 generated values (Monte Carlo simulations). For the three cases, 20 simulations were realized for each model. Vertical lines indicate the range of the simulated results, with associated coefficients of variation.

The precision on the correlation coefficient equals 95% when the simulation is realized with 100 generated values, and 99% with 1 000 generated values. Similar tests carried out on the other parameters

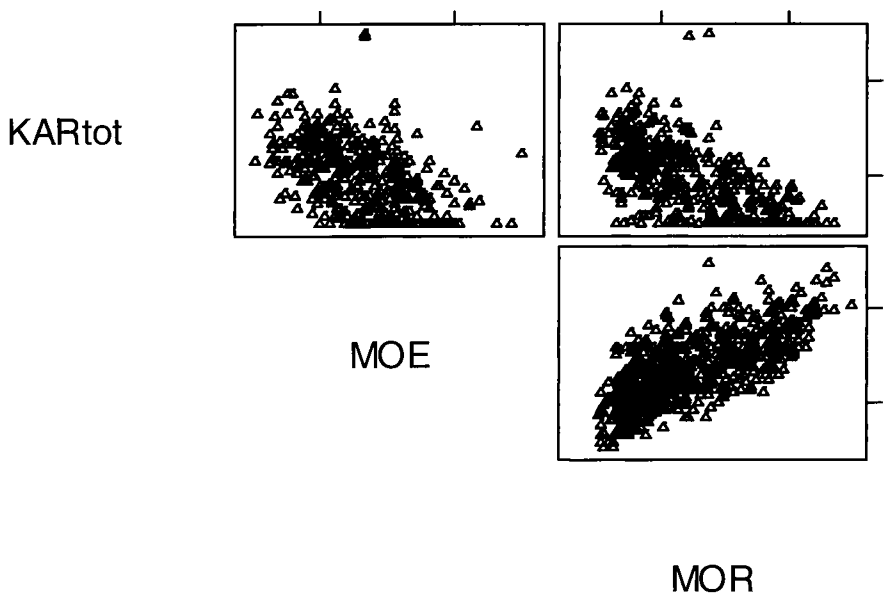


Fig 2. Correlation matrix plot for KAR_{tot}, MOE and MOR. The intervals of variation for each variable are given in table I.

Table IIIa. Correlation matrix for the experimental values (uncorrected coefficients).

	Density	*MOE*	*MOR*
MOE	0.748	1	1
MOR	0.625	0.775	1
KAR	− 0.431	− 0.552	− 0.655

KAR: knot area ratio; see table I for other abbreviations.

Table IIIb. Coefficients of correlation for the experimental 'normalized' values.

	Density	*MOE*	*MOR*
MOE	0.748		
MOR	0.624	0.773	
KAR	− 0.431	− 0.555	− 0.644

See table I for abbreviations.

resulted in a similar precision. Consequently, the dispersion of the values calculated by the different models decreased when the number of generated values increased. For this reason, the next simulations will be made with 1 000 values.

Model comparison

Simulated histograms of MOR obtained from one simulation are represented in figure 4 for each correlation model. We used different statistics to estimate the goodness-of-fit of simulated distributions with respect to the experimental data. The Kolmogorov-Smirnov statistics do not provide stable information and we cannot afford to

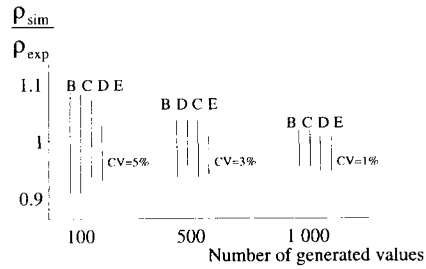


Fig 3. Influence of the number of generated values in Monte Carlo simulations on the predicted coefficient $\rho_{MOE/MOR}$. CV: coefficient of variation COV (%).

decide which simulated pdf is the most appropriate. The relative goodness-of-fit of the simulated MOR histograms by the models A, B, E (complete model) to the experimental histogram were therefore evaluated by χ^2 statistics. The results are presented in table V. The best fit was obtained for the model MOE/MOR with a significance level of 70%. We would expect the highest significance level with the complete model, using three NDE. This result might reveal some limits in the simulation procedure, especially if the degree of uncertainty of one predictive variable is dominant (ie, the COV of this variable is larger than the COV of other variables). In our case, the large variations of KAR_{tot} and the choice of a normal distribution for this variable may affect significantly the output distribution form.

We tested the efficiency of the different models in predicting the mean and 5% frac-

Table IV. Regression equations of the MOR with different combinations of nondestructive estimators (NDE).

	MOE a_1	*Density* a_2	*KAR* a_3	*Intercept*	ρ^2
MOR	7.2	–	–	− 30	0.6
	6.3	0.05	–	− 49.5	0.61
	5.4		− 0.037	− 2.8	0.65
	4.5	0.047	− 0.037	− 20.9	0.66

Modulus of rupture (MOR) = $a_1 x_1 + a_2 x_2 + a_3 x_3$ + intercept.

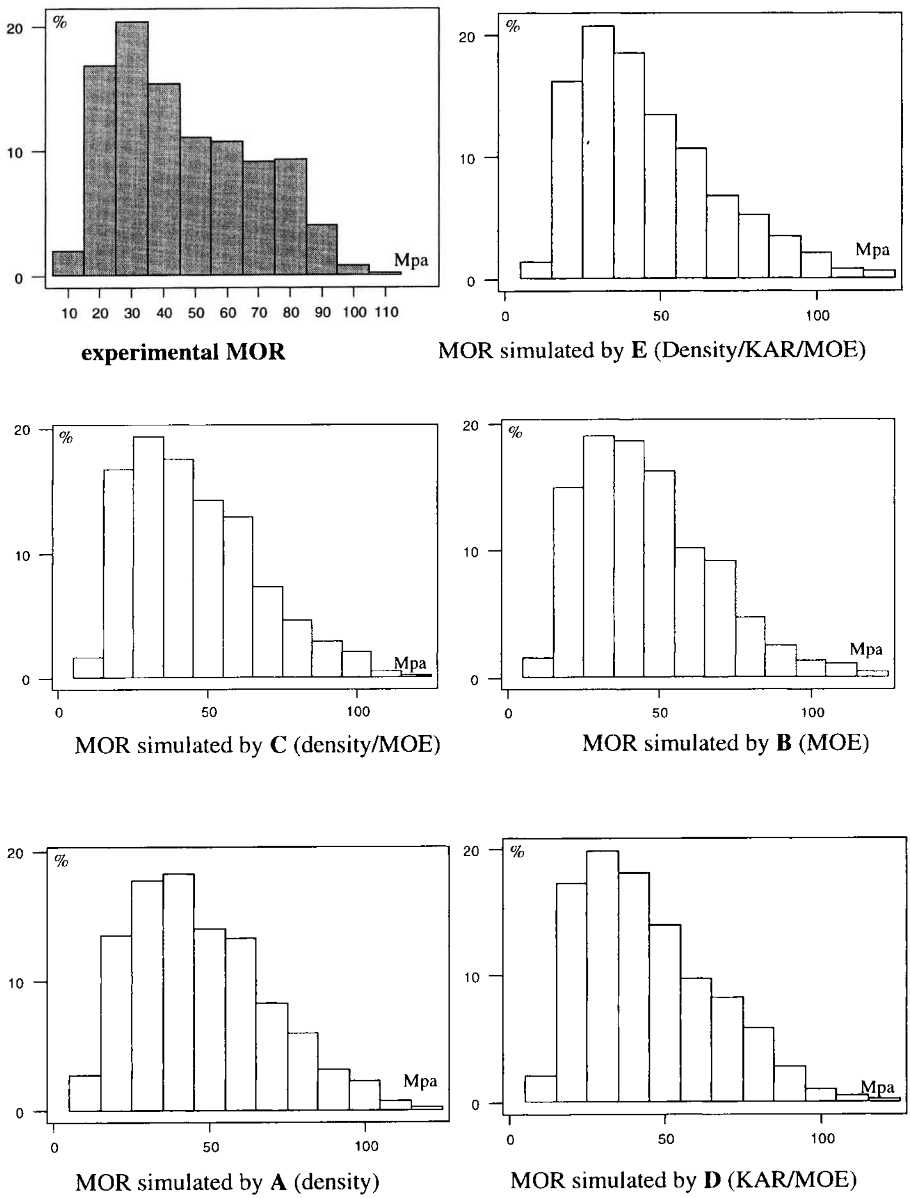


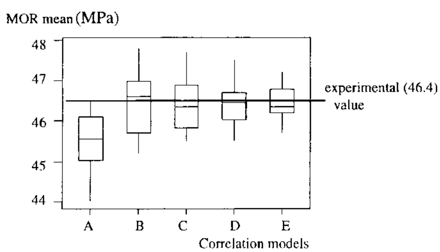
Fig 4. Experimental and simulated histograms of MOR using different correlation models.

Table V. Goodness-of-fit analysis (χ^2 statistics) on the simulated distributions: comparison with the experimental probability density functions (pdf).

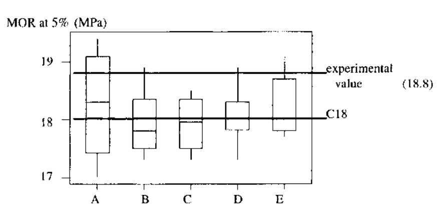
	Density/MOR	*Density/KAR/MOE/MOR*	*MOE/MOR*
χ^2 statistics	30%	50%	70%

tile of MOR. Results are presented in figure 5a and b. Box plots represent the dispersion obtained from 20 simulations of 1 000 values each.

Model A (density-based model) tends to underestimate the average MOR, whereas the other models give a good prevision of this parameter. According to the previous regression analysis, models D and E present the lowest degree of uncertainty (residual variance), and therefore they both seem to provide more reliable results (numerical stability). On the other hand, the prediction of the characteristic value is below the expected experimental value for all models (fig 5b). This is probably because the goodness-of-fit analysis for the choice of theoretical pdf, especially for MOR, has been performed on the complete experimental distribution. The simulation results could be improved by considering only the lower tail of distributions in the statistical modeling of data.



(a)



(b)

Fig 5. **(a)** Comparison of average MOR values calculated with the different correlation models. Box plots indicate the dispersion on the predicted average for 20 simulations of 1 000 generated values. **(b)** Comparison of MOR characteristic values (5% fractile) calculated with the different models. Horizontal lines indicate the experimental characteristic value (estimated at 18.8 MPa) and the target value for the C_{18} strength class.

Validity of the correlation models for fast grown trees

The general correlation model can be used to simulate the characteristics of one particular class in the population of beams. This was done for the youngest trees (age 1), using the corresponding statistical parameters (mean and standard deviation) of D_{12} (table I). Simulated pdf of MOR are presented in figure 6. The χ^2 statistics show that all correlation models predict fairly well experimental data. However, the simulated average MOE is higher than the expected average obtained from the database. This result indicates that the proportion of juvenile wood in beams mainly affects the prevision of elastic properties for given density and visual grade.

APPLICATION OF MULTIVARIATE MODELS TO THE GRADING OF TIMBER

The previous analysis has shown some efficiency of correlation models in predicting the distributions of concomitent properties, and the study of the influence of one or sev-

Table VI. Optimum classification of the database with a hierarchical clustering analysis. Characteristics of the different classes.

	Database	*Class A*	*Class B*	*Class C*	*Class D*
Ring width (mm)	3.7	2.6*	3.2*	3.8*	5.3*
Density (kg/m^3)	562	625*	590*	541*	502*
KAR (%)	24	5	17	30	38
MOE (GPa)	10.2	14*	11.4*	9*	7.2*
MOR (MPa)	42.3	71*	51.3**	32*	24*
Percentage	100%	12%	37%	36%	16%

* Significant at the 0.1% level; ** significant at the 1% level.

eral NDE on the variability of estimated mechanical characteristics, especially strength properties. On the other hand, if the goodness-of-fit analysis does not provide a good prediction of the tails of distributions no conclusions can be drawn from simulations concerning the extreme values of the predicted variables, and only the average values can be predicted.

Regardless of this limiting aspect, statistical models can have applications in the grading of lumber. Let us consider for instance a population composed of N values of concomitent variables: density, KAR, MOE, MOR and identification parameters such as age of trees, position of logs and growth rate. Within this sample an optimum classification would divide the population in N_{30} beams entering the C_{30} class, N_{22} beams entering the C_{22} class and so on, so that:

$$\frac{\Sigma N_{xx} + N_{rej}}{N} = 1$$

The optimal classification should lead to a statistical homogeneity of classes, that is, the within-class heterogeneity will be lowest. This can be obtained for instance from multiple correspondance analysis.

Hierarchical clustering on the database

A hierarchical clustering (Ward metries) performed on the factors obtained from a multiple correspondance analysis on the previous database led to a classification of the sample into four homogeneous classes that can be described as follows: class A: high grade lumber (defect free pieces with high density and mechanical properties), class B: medium grade lumber with a low KAR and high mechanical properties, class C: low grade lumber and class D: very poor grade (pieces with a high KAR and very high growth rate).

The percentage of beams entering each class, and the average characteristics, are presented in table VI. With this classification it is found that the whole sample would at least enter the C_{18} strength class (see also fig 5b), and 60% would be classified in the C_{30} class. Visual grading according to the NFB 52-001 standard for maritime pine is indeed more severe: 38% of the beams are excluded from the C_{18} class, whereas almost 25% of the rejected pieces belong to class B or class A. Mechanical stress grading using one NDE of strength should be closer to the optimal classification of the population of lumber.

Simulation of grading through correlation models

Correlation models can be used to estimate the percentage of pieces that will enter different strength classes, using the information obtained from one NDE, and estimate the distributional characteristics of lumber within each class – given, for instance, a population of visually graded beams for

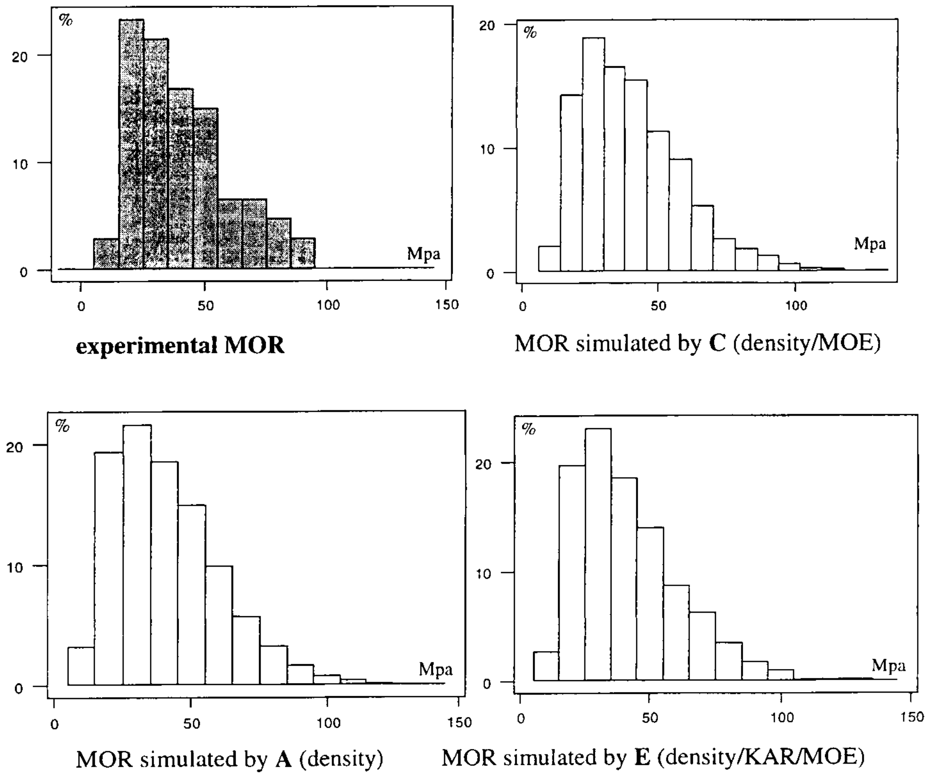


Fig 6. Experimental and simulated histograms of MOR for the first age class (young trees) using density as predictor.

which the mean and standard deviation of density have been estimated by quality control on a subsample. The classification of beams in different strength classes can be calculated from correlations, and the global 'quality' of the population can be assessed before grading. If the estimated quality is poor, only the pieces entering the C_{18} class will be selected using visual criteria. However, if the population is composed of high quality lumber, on-line grading through density measurements or stress grading will optimize the classification.

Such simulations have been made using the maritime pine database, and the results are shown in table VII. Three populations are compared: low, medium and high density, with around 10% variations around the average value for each population. Concerning the low density population, we found that 32% of the sample could enter the C_{30} class if a stress grader is used with a limiting value of MOE = 10.4 GPa and only 10% if D_{12} is used as a sorting variable. Note also that grading for the C_{22} strength class increases the percentage of pieces that will be excluded from structural uses. The density characteristics of the population affects the percentage of high grade lumber: with a stress grading system this percentage increases from 30 to 65%, and with density measurements it increases

Table VII. Simulation of grading with the correlation model, using one nondestructive estimator (NDE) (D_{12} or MOE), and three different populations of lumber characterized by the statistical parameters of density (mean and standard deviation).

*Population characteristics (density *)*	*NDE: MOE (GPa)*			*NDE: Density (kg/m³)*		
	C_{30}	C_{22}	C_{18}	C_{30}	C_{22}	C_{18}
m = 554 SD = 51	50% min = 10.4	0	37% min = 8.8	37% min = 565	0	50% min = 500
	0	82% min = 7.8	0	0	70% min = 520 3	0
	0	0	99% min = 5.6	0	0	98% min = 450
m = 535 SD = 43	32% (10.4)	15% (9.6)	0	10% (585)	35% (540)	0
	32% (10.4)	0	56% (7.5)	10% (585)	0	70% (500)
	0	73% (8.3)	0	0	50% (535)	6% (530)
	0	0	93% (7)	0	0	80% (500)
m = 575 SD = 51	65% (9.8)	0	27% (7.8)	53% (570)	0	45% (560)
	0	93% (7.6)	0	0	90% (500)	0
	0	0	99% (4.9)	0	0	99% (420)

The table gives the percentage of graded pieces and the lower bound for the NDE, according to various strategies in each situation. * Average value (m) and standard deviation (SD).

from 10 to 53%. This result can be used in previsional analysis of wood quality with indirect information on the biological origin of logs (juvenile or mature wood).

CONCLUSION

Statistical modeling of beam strength from correlation models has been used to simulate grading procedures for various populations of maritime pine lumber. The percentage of graded beams, and the distributional characteristics of each subsample, can be estimated from the simulations.

The development of NDE techniques in lumber will provide useful information in the development of statistical multivariate models for various species. The statistical approach can be used to increase the reliability of such techniques for industrial applications. However, the use of visual parameters, such as the KAR_{tot}, should be considered with prudence in correlation analysis. The actual effect of knots on strength cannot be easily reduced to a single parameter, and further research is needed in this area. Recent developments in modeling the strength of lumber consider for instance the lengthwise distribution of defects in beams, which provides additional information on the minimal distance between two weak zones. Such parameters can induce stress concentration effects and, in turn, modify the actual strength and the failure mode.

As far as high grade lumber is concerned, multivariate models can be used for the elasticity and strength modeling of glulam or LVL (laminated veener lumber) products, using statistical information on the constituants. Multivariate analysis can also be extended to other mechanical properties, such as the creep behavior of wood members. This aspect is the main objective of future research, with expected applications in reliability analyses and design of wood-based products.

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