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A tree crown ratio prediction equation for eucalypt plantations

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Abstract – Based on a data set from spacing trials and permanent plots of *Eucalyptus globulus* Labill., several nonlinear equations for crown ratio prediction (based on exponential, logistic, Richards and Weibull functions) were tested. The total data set was used to fit and select the equations. The equations were evaluated in terms of measures of fit and prediction ability: adjusted-R^2, residual mean square, sum of PRESS residuals and sum of absolute PRESS residuals. The normality of the studentized residuals was analyzed using normal QQ plots. The presence of heteroscedasticity associated with the error term was checked by plotting the studentized residuals against the predicted values. The significance of the estimated parameters was verified. Model error was characterized in terms of bias and precision. The Richards function was selected. This equation is age and density dependent, reflecting the importance of intertree competition; an initial tree dimension and a measure of stand productivity were also required as explanatory variables.

crown ratio / tree model / plantations / *Eucalyptus globulus* Labill.

Résumé – Équation de prédiction du rapport entre longueur du houppier et hauteur totale de l'arbre pour des plantations d'eucalyptus. À partir d'un ensemble de données d'essais d'espacement et de parcelles permanentes *d'Eucalyptus globulus* Labill, différentes équations de prédiction du rapport entre longueur du houppier et hauteur totale de l'arbre (basées sur les fonctions exponentielle, logistique, Richards et Weibull) sont testées. Les données sont utilisées pour l'estimation et la sélection des équations de prédiction. L'évaluation des équations est basée sur des mesures d'ajustement et de capacité de prédiction : R^2-ajusté, carré moyen des résidus, addition des résidus PRESS et addition des résidus PRESS absolus. La normalité des résidus est analysée par le graphique QQ normal. La présence d'hétéroscedasticité associée à l'erreur est analysée par le graphique des résidus versus les valeurs prédites. La signification des paramètres estimés est vérifiée. L'erreur du modèle est caractérisée en terme de biais et de précision. La fonction de Richards est sélectionnée. Cette fonction est dépendante de l'âge et de la densité, exprimant ainsi l'importance de la compétition entre les arbres ; la dimension initiale de l'arbre et une mesure de la productivité du peuplement interviennent aussi comme variables explicatives.

rapport longueur du houppier-hauteur totale / modèle individuel / plantations / *Eucalyptus globulus* Labill.

1. INTRODUCTION

Crown dimensions can be important components of forest growth and yield models, and are used in many tree and crown level growth–modelling systems [2, 3, 17, 19, 21, 25]. For instance, tree crown parameters can be considered when simple competition indices are not able to adequately predict recovery from competition when a competitor is removed (e.g. by thinning) [23]; tree crown parameters have been used as predictor

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variables in diameter and height growth equations [e.g. 3, 9, 25]; stand crown parameters have also been used to distinguish different stages of stand development [18].

A tree's crown reflects the cumulative level of competition over time [8]. Crown ratio and crown length reflect the potential of a released tree to use available resources such as increased growing space [3]. The lack of data and the difficulty of accurately measuring the height to the live crown base – more pronounced in species with asymmetric crowns – may justify the relatively little research done on modeling crown parameters [17]. Crown dimension modeling is highly dependent on the accuracy of total height and height to the live crown base data and/or equations. Predictions of tree crown ratio have been based on allometric relations between stand and tree variables [1, 5, 6, 7, 25, 26]. Crown ratio can be predicted directly from tree and/or stand variables [e.g. 6] or indirectly from estimates of the height to the live crown base.

The purpose of the present work was to develop a tree crown ratio prediction equation, that serves as a component of a tree model for the first cutting cycle of *Eucalyptus globulus* Labill. plantations, located in the north and central coastal regions of Portugal [18]. This equation will be used to determine the stand average crown ratio, an indicator of different competition stages during stand development.

2. DATA

Eucalyptus globulus Labill. was introduced in Portugal 150 years ago. It is a fast growing species, mainly being used by the pulp industry. The trees are planted at final density, as thinning and pruning practices are not usual during the first cutting cycle. The stands are intensively managed as a short rotation coppice system in which the first cycle of planted seedlings (single stem) is followed by 2 or 3 coppiced stands, with an average cutting cycle of 10–12 years.

Data from permanent plots, three spacing trials and a fertilization and irrigation experiment of eucalypt were used. The principal criterion for the selection of these plots was the availability of tree measurements of the height to the live crown base (*table I*). In some of the plots, the measurements were made on felled trees, but in most of them they were gathered with a hypsometer. The base of the live crown was defined as the point of insertion of the lowest live branch in at least three of the four horizontal quadrants defined around the stem of the tree. However, this definition is, in practice, subjective; when possible, this subjectivity was minimized by maintaining the same field crew in subsequent measurements and checking, directly in the field, the values of the height to the live crown base, comparing them with the values of the last measurement.

This data set includes 10 and 36 plots respectively from the Quinta do Paço and the Vilar de Luz spacing trials [18], two control plots and two fertilized plots of the fertilized and irrigated trial [12] and 18 permanent plots. Data from felled trees in 10 plots of the Alto do Vilão spacing trial and two permanent plots, described by Pedro [11] and Tomé [20], were also used. Individual tree leaf area was estimated [13]. In the stands where the height to the live crown base of all the trees were measured, the leaf area index (LAI) was defined as the ratio between the total leaf area and the plot area. When only a sample of trees was measured, LAI was calculated by multiplying the mean leaf area by the number of trees in each plot and dividing by the plot area.

From the available measurements of individual trees, those from border trees, trees without simultaneous measurements of total height, height to the live crown base and diameter and trees with height and/or diameter imperfections were eliminated.

3. METHODS

3.1. Candidate models

A crown ratio equation must be bounded so that the crown ratio prediction values lay between 0 and 1. Many authors have based crown ratio equations on the logistic function [6, 14] or the exponential function [4, 5, 7]. In this work, the Richards function and the cumulative distribution of the Weibull function were also tested (*table II*).

The linear function X (see *table II*) was expressed as a combination of age, tree dimension (diameter, height, height/diameter ratio), stand density (number of planted trees or live trees ha^{-1}, basal area), maximum tree dimension (diameter), mean tree dimension (diameter, dominant diameter) and site productivity (dominant height, site index). The inverse of each of these variables was also tested. The number of variables was restricted by the presence of only one from each group. Each function was tested in different versions defined by the linear function of tree and stand variables (X). As an exploratory analysis, an all-possible-regression algorithm, with crown ratio as dependent variable, was used to select combinations of variables to express crown ratio. This selection was based on measures of multiple linear regression quality and prediction ability: adjusted-R^2, residual mean square, sum of PRESS residuals and sum of absolute PRESS residuals [10]. The presence of

Table I. Characterization of the plots used for the definition of the crown ratio prediction equation.

		spacing trials				permanent plots		
		AV	QP	VL	FR	PP	Pedro (1991)	Tomé (1997)
plot characteristics								
plot area (m^2)		1584–2464	648–2916	470–2475	1089	243–780	2916	1006
spacing (m×m)		3×2–5×4	2×1–3×3	2×1–4×4	3×3	1.7×1.7–3.3×3.2	3×3	3×3
site index		20.4–23.7	25.7–28.4	19.7–25.8	23.3–28.1	12.6–26.8	23.3	22.6
number of remeasurements		1	4	7	3	4	1	1
age (years)	minimum	17.9	4.6	1.8	2.0	1.3	5.3	6.7
	mean	17.9	6.1	2.8	3.1	4.6	5.3	7.3
	maximum	17.9	7.6	4.8	4.3	8.3	5.3	7.9
n° trees/plot	minimum	5	39	56	79	17	324	18
	mean	5	50	61	82	38	324	18
	maximum	5	61	65	86	85	324	18
d (cm)	minimum	7.2	1.2	0.2	4.2	0.4	3.2	8.1
	mean	20.7	11.9	6.2	11.3	7.6	12.6	14.0
	maximum	33.2	23.9	22.8	19.7	22.1	22.3	18.7
h (m)	minimum	10.0	3.0	1.1	4.1	1.4	5.7	14.7
	mean	23.6	16.8	7.8	11.0	9.4	13.3	17.6
	maximum	30.5	26.8	20.0	16.9	26.5	19.5	22.2
hbc (m)	minimum	7.8	2.7	0.0	0.1	0.2	1.3	4.5
	mean	17.7	10.5	2.6	3.4	3.9	3.9	6.6
	maximum	25.6	19.5	13.6	7.2	14.7	7.6	10.0
cl (m)	minimum	0.9	0.0	0.2	3.2	0.4	2.5	7.8
	mean	5.9	6.4	5.6	7.6	5.5	9.4	11.0
	maximum	16.5	14.5	12.9	12.4	16.0	15.2	14.9
cr	minimum	0.05	0.00	0.08	0.46	0.11	0.34	0.50
	mean	0.25	0.37	0.72	0.71	0.59	0.70	0.62
	maximum	0.60	0.80	1.00	0.99	0.98	0.90	0.75
la (m^2)	minimum	4.05	0.29	0.01	4.31	0.03	2.48	10.85
	mean	23.20	17.01	10.27	29.41	14.67	38.13	36.43
	maximum	64.49	71.32	100.18	71.85	117.91	107.28	62.36
LAI	minimum	1.21	1.79	0.13	1.73	0.18	3.62	3.26
	mean	1.68	2.68	1.60	3.23	2.12	3.62	3.26
	maximum	2.15	3.76	3.93	4.70	3.80	3.62	3.26

AV, QP and VL, Alto do Vilão, Quinta do Paço and Vilar de Luz spacing trials, respectively; FR, fertilization and irrigation trial; site index, mean height at base age 10 years; d, tree diameter at breast height; h, total tree height; hbc, tree height to the live crown base; cl, tree crown length; cr, tree crown ratio; la, tree leaf area; LAI, leaf area index.

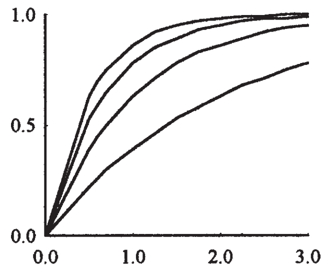
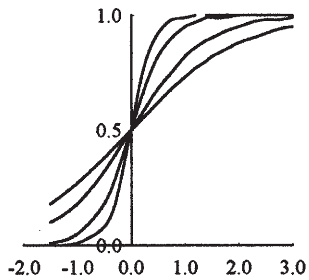
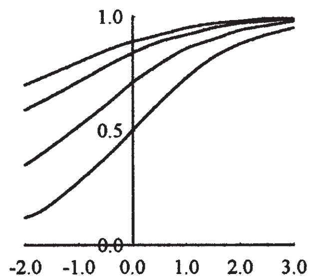
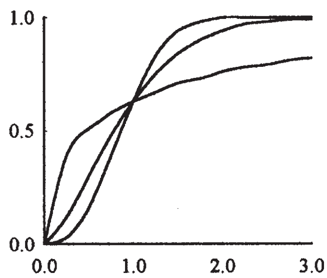
colinearity was analyzed on the basis of the values of the variance inflation factors (VIF); values up to 10 were accepted [10].

3.2. Model fitting and selection

Model fitting and evaluation are important parts of model building. In this work, the total data set was used to fit and select the tree crown ratio equations. The evaluation was based on the prediction errors or PRESS residuals.

Different versions of the exponential, logistic, Richards and Weibull functions were fitted; the parameter estimation of these nonlinear functions was based on the least squares method associated with the PROC NLIN procedure of the SAS/STAT [16]. The modified

Table II. Functions tested and restrictions imposed to the parameters on the prediction of tree crown ratio.

function	restrictions	
exponential $y = A\left[1 - c\,e^{-kX}\right]$	*A*, *c*, *k* = 1	
logistic $y = \dfrac{A}{1 + c\,e^{-kX}}$	*A*, *c*, *k* = 1	
Richards $y = \dfrac{A}{\left[1 + c\,e^{-kX}\right]^{1/m}}$	*A*, *c*, *k* = 1 *m* = 6 *	
Weibull $y = \mathrm{A}\left[1 - c\,e^{-kX^{w}}\right]$	*A*, *c*, *k* = 1 *w* = 10 *	

y, tree crown ratio; X, linear function of tree and stand variables; *A*, asymptote; *c*, *k*, *m* and *w*, function parameters; *, defined on point 4.2. of this work.

Gauss-Newton iterative method was applied in model fitting. The PROC MODEL procedure of the SAS/ETS [15] was used to analyze the colinearity between the variables and to ensure that the solution was global rather than local. Multicollinearity was assessed in terms of the condition number of the correlation matrix; when this exceeded 1000, the effect of multicollinearity was considered serious and the model discarded [10].

The versions of each function were identified and were evaluated in terms of measures of fit and prediction ability: adjusted-R^2, residual mean square, mean of PRESS residuals and mean of absolute PRESS residuals [10]. The normality of the studentized residuals was analyzed using normal QQ plots. The presence of heteroscedasticity associated with the error term of the models was checked by plotting the studentized residuals against the predicted values. The heteroscedasticity was only checked graphically because the frequent non-normality of the studentized residuals makes the use of statistical tests impracticable [24]. Both the significance and the stability of the parameters estimated were ensured based on the asymptotic t-statistics.

3.3. Model evaluation

The evaluation was based on the prediction errors or PRESS residuals that indicate the predictive ability of the equations by cross validation [10]. This entails omitting each observation in turn from the data, fitting the model to the remaining observations, predicting the response for the omitted observation and comparing the prediction with the observed value: $y_i - \hat{y}_{i,-i} = e_{i,-i}$ ($i = 1,2,\ldots, n$). The PRESS residuals are true prediction errors with $\hat{y}_{i,-i}$ being independent of y_i. Each candidate equation has n PRESS residuals associated with it, and the PRESS (Prediction Sum of Squares) statistic is defined as [10]:

$$\text{PRESS} = \sum_{i=1}^{n} \left(y_i - \hat{y}_{i,-i}\right)^2 = \sum_{i=1}^{n} \left(e_{i,-i}\right)^2.$$

The accuracy of the selected functions, in terms of both bias and precision, was analyzed. Bias and precision were assessed through histograms of the PRESS residuals and computation of the mean of the PRESS residuals (bias) and the mean of the absolute PRESS residuals (precision). Average model bias measures the error when several observations are combined by totaling or averaging, and mean absolute difference measures the average error associated with a single prediction [22]. The interquantile range of the PRESS residuals (Q99-Q1) was also computed as a measure of precision. Plots of observed values over predicted values were also analyzed.

The modelling efficiency was computed; this statistic provides a simple index of performance on a relative scale, where 1 indicates a perfect fit, 0 reveals that the model is no better than a simple average, and negative values indicate a poor model indeed [22].

4. RESULTS AND DISCUSSION

In the data set, the lack of crown ratio data in stands more than 8 years old was evident (*figure 1*). In fact, height to the live crown base is not measured in current forest inventory; most of the crown ratio data was obtained from spacing trials or permanent plots integrated in a special schedule of measurements. The large range of the crown ratio values in each age reflects the effect of stand density and site productivity; this variation was also observed when the data was analyzed at stand level, in spite of the fact that these stands are even-aged monocultures.

4.1. Candidate models

The all-possible-regression algorithm applied to the total data set resulted in variation inflation factors (VIFs) greater than 10 with combinations of 5 and 6 variables. As a consequence, the number of independent variables

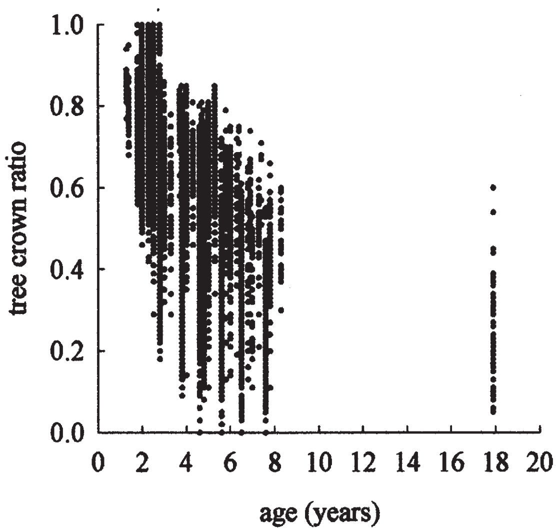


Figure 1. Relation between tree crown ratio and age on the total data set (nobs = 19041).

used in the X function was restricted to 3 or 4. The combinations of variables that showed a good performance in the all-possible-regression algorithm were tested in order to select the best nonlinear model – exponential, logistic, Richards or Weibull function.

4.2. Model fitting and selection

Table III presents the selected versions of the exponential and logistic functions. Both were age dependent (with best results with the inverse of age); the number of live trees, instead of the basal area, expresses the stand density; and the initial tree dimension was an important variable to define the tree crown ratio. In the logistic function the productivity was best expressed by the dominant height.

Convergence problems were detected in the fitting of the Richards and Weibull functions when the parameters were not restricted. To estimate *m* and *w* parameters, associated respectively with the Richards and the Weibull functions, it was decided to test a set of fixed values of these parameters. The parameter estimates of the best versions of the logistic and exponential functions were used as initial values. The *m* and *w* corresponding to the smallest residual sum of squares were selected: Richards function, $m = 6$; Weibull function, $w = 10$.

Four functions were selected, designated as E: exponential, L: logistic, R: Richards and W: Weibull (*table III*).

The hypothesis of normality of the studentized residuals was rejected for all functions. However, the normal QQ plots did not present strongly stressed asymmetries. These plots were also used to detect outliers; when identified as measurement errors, handwriting or computation typing errors they were corrected.

Figure 2 shows the graphic relationship between the studentized residuals and the crown ratio estimates obtained with the E, L, R and W functions. For the L, R and W functions, a systematic variation of the residuals was not observed, although a greater dispersion associated with the smaller predicted values was evident. Lower crown ratio values characterized old stands (where the measurements of the total height and the height to the live crown base are more difficult and less accurate) or, in the same stand, the suppressed trees. The E function suggested a slight decreasing tendency.

Figure 2 allows the identification of plots or sets of plots characterized by an abnormal relation between the observed crown ratio values and the stand characteristics (e.g. age, site index and density). All these plots were checked, and maintained when the veracity of the crown ratio values was proven. The early measurements of one of the spacing trials – Vilar de Luz – contributed many of these observations: trees of the wider spacings did not

Table III. Tree crown ratio equations selected: exponential (E), logistic (L), Richards (R), Weibull (W) ($n = 19\,041$).

function	adj-R^2	RMS	min.	max.
E: $cr = 1 - e^{-\left(3.82724\frac{1}{t} - 0.08693\frac{N}{1000} - 0.01551\,ddom + 0.02969\,d\right)}$	0.74	0.011	–0.08	0.93
L: $cr = \dfrac{1}{1 + e^{-\left(-1.05195 + 6.01605\frac{1}{t} - 0.13592\frac{N}{1000} - 0.04759\,hdom + 0.07236\,d\right)}}$	0.76	0.010	0.16	0.96
R: $cr = \dfrac{1}{\left[1 + e^{-\left(-5.76111 + 12.33413\frac{1}{t} - 0.27179\frac{N}{1000} - 0.17543\,hdom + 0.20559\,d\right)}\right]^{1/6}}$	0.77	0.010	0.23	0.99
W: $cr = 1 - e^{-\left(-0.91024 + 0.36370\frac{1}{t} - 0.01036\frac{N}{1000} - 0.00405\,ddom + 0.00502\,d\right)^{10}}$	0.77	0.010	0.17	0.99

t, age; *N*, number of trees ha^{-1}; *d*dom, dominant diameter (cm); *h*dom, dominant height (m); *d*, tree diameter at breast height (cm); RMS, residual mean square; min., minimum tree crown ratio predicted value; max., maximum tree crown ratio predicted value.

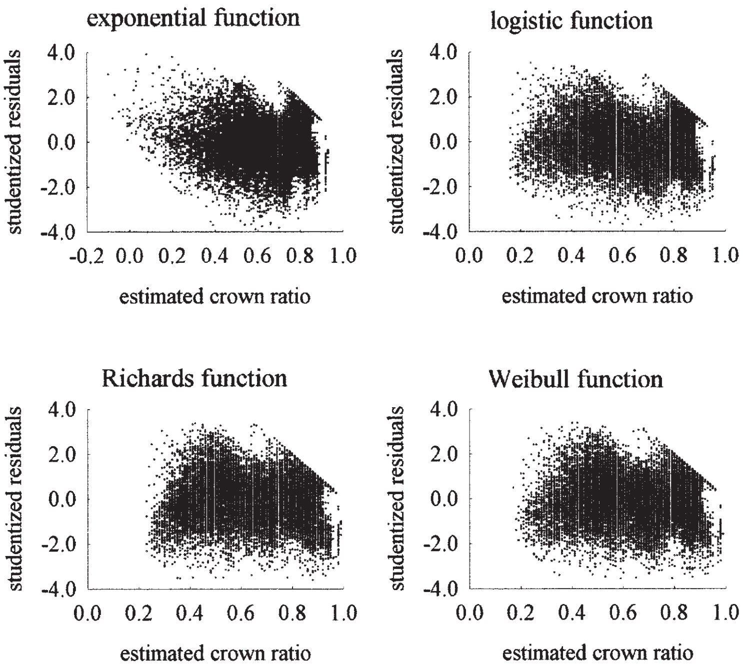


Figure 2. Graphical relationship between studentized residuals and crown ratio values estimated with the exponential, logistic, Richards and Weibull functions.

yet show a rise of the base of the crown and the crown ratio values equalize the asymptote of the functions. Different densities of the plots of this trial were reflected in the visual aspect of the limit line that characterized each one of the graphs in *figure 2*.

The exponential function estimated negative values, which were out of the range of admissible values (*table III*). *Table II* presents the parameter restrictions used to oblige the crown ratio to always be positive. However, for the exponential function, X (a linear combination of tree and stand variables) should also be positive. In the data set, 9 predicted crown ratios (in 19 041 observations) were negative as a consequence of a negative X; these points were identified as corresponding to trees growing in very dense plots (5 000 trees ha^{-1} at plantation) with high values of dominant diameter or to trees with small diameters growing in stands with high values of dominant diameter.

In spite of the fact that the crown ratio values predicted by the three functions laid between 0 and 1, the logistic function predicted the lowest maximum values (0.96) which, according to the characteristics of the total data set, seemed to underestimate the observed values; the Richards function predicted the highest minimum values (0.23), which seemed to overestimate the observed values (*table III*). In the total data set, 10.5% of the real observations were greater than 0.99; null crown ratio values were observed corresponding mainly to near-to-death-trees of the closer spacings of the Quinta do Paço trial. The mean crown ratio of the old stands, represented by the Alto do Vilão spacing trial, was 0.25 and the minimum values predicted by both functions were not inferior to that limit, suggesting a good adherence by both young and old stands.

According to the apparent heteroscedasticity associated with the error term of the exponential function and the ability of this function to predict negative crown ratio values, only the logistic, Richards and Weibull functions were proposed for the evaluation task.

4.3. Model evaluation

Table IV presents the mean values of the measures of precision and bias associated with the three functions analyzed as well as the correspondent modelling efficiency. Bias shown by each one of the functions, logistic, Richards and Weibull, was negligible. The Richards and Weibull functions were simultaneously more precise and less biased; these functions were associated with the highest values of modelling efficiency.

The graphs of observed *versus* predicted crown ratio values, for the three functions, did not disclose a linear relation (*figure 3*). However, the dispersion observed

Table IV. Evaluation of the logistic (L), Richards (R) and Weibull (W) functions.

function	MPRESS	MAPRESS	PRESS	ME	Q99-Q1
L	0.0029	0.083	199.1	0.76	0.234–(–0.246) = 0.480
R	0.0002	0.081	193.8	0.77	0.231–(–0.248) = 0.479
W	0.0008	0.082	195.2	0.77	0.234–(–0.248) = 0.478

MPRESS, mean PRESS residuals; MAPRESS, mean absolute PRESS residuals; PRESS, PRESS statistic; ME, modelling efficiency; Q99, quantil 99; Q1, quantil 1.

around the line ($y = x$) for crown ratio values between 0.25 and 0.85 was well balanced.

The results of the analysis of the accuracy by age, site index and planting density classes associated with the three crown ratio functions is shown in *table V*.

The decrease of precision with increasing age was evident for all the functions. The reduced number of crown ratio values associated with ages greater than 8 years and the low accuracy associated to the total height and height to the live crown base measurements for these ages can contribute to the observed tendency. The first age class was simultaneously the least biased and the most precise. The Richards and Weibull functions were the most accurate functions for ages lower and higher than 8 years, respectively; this fact seemed to confirm the tendency previously observed.

The analysis by planting density classes showed that all functions were more accurate in the densest class. An increase of accuracy associated with an increase of site index was observed with all the functions; globally, the Richards function was the most precise for site index classes greater than 16; the Weibull function was the least biased.

From the analysis of *table V*, it was evident that:

– there is only a small number of observations of the height to the live crown base in stands older than 8 years; the extrapolation ability of the crown ratio prediction equation will be restricted by the range of the variables that characterize the total data set;

– a strict relation exists between the results of the evaluation task and the quality of the data that could be affected by the subjectivity inherent to the definition of the height to the live crown base;

– the Richards and Weibull functions seem unable to estimate, for this data set, crown ratio values less than 0.23 and 0.17, respectively.

5. CONCLUSION

Based on the analyses described in this paper, the Richards function is recommended for a tree crown ratio prediction equation of eucalypt stands in Portugal:

Richards function:

$$cr = \frac{1}{\left[1 + e^{-\left(-5.76111 + 12.33413\, 1/t - 0.27179\, N/1000 - 0.17543\, h\mathrm{dom} + 0.20559\, d\right)}\right]^{1/6}}$$

where t, age (years); N, number of trees per hectare; $h$dom, dominant height (m); d, diameter at breast height (cm).

This function is age and density dependent, reflecting the importance of competition; age was expressed by its

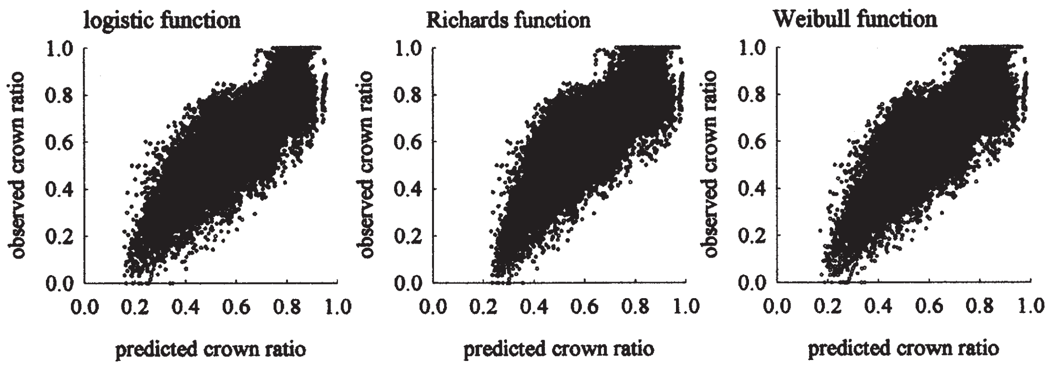


Figure 3. Relation between the observed crown ratio and the crown ratio values estimated with the logistic, Richards and Weibull functions.

Table V. Mean of the PRESS residuals and mean of the absolute PRESS residuals by age, site index and density at plantation classes for the logistic (L), Richards (R) and Weibull (W) functions.

	Mean of PRESS residuals				Mean of absolute PRESS residuals			
Age classes								
Function	t ≤ 4	4 < t ≤ 8	8 < t ≤ 12	t > 12	t ≤ 4	4 < t ≤ 8	8 < t ≤ 12	t > 12
nobs	13468	5485	37	51	13468	5485	37	51
L	–0.0010	0.0123	0.1729	–0.1008	0.082	0.084	0.174	0.154
R	–0.0015	0.0048	0.1331	–0.1330	0.079	0.086	0.135	0.166
W	–0.0018	0.0071	0.1333	–0.0876	0.079	0.086	0.140	0.140
Site index classes								
Function	SI ≤ 16	16 < SI ≤ 20	20 < SI ≤ 24	SI > 24	SI ≤ 16	16 < SI ≤ 20	20 < SI ≤ 24	SI > 24
nobs	221	1501	9085	8234	221	1501	9085	8234
L	–0.1285	0.0010	0.0115	–0.0026	0.134	0.090	0.087	0.075
R	–0.1466	–0.0098	0.0097	–0.0045	0.153	0.086	0.084	0.075
W	–0.1367	–0.0028	0.0102	–0.0052	0.145	0.090	0.084	0.075

	Mean of PRESS residuals			Mean of absolute PRESS residuals		
Density at plantation classes						
Function	Npl ≤ 1111	1111 < Npl ≤ 1667	Npl > 1667	Npl ≤ 1111	1111 < Npl ≤ 1667	Npl > 1667
nobs	4589	6287	8165	4589	6287	8165
L	0.0072	0.0006	0.0024	0.085	0.084	0.080
R	0.0043	–0.0003	–0.0017	0.082	0.081	0.081
W	0.0066	–0.0014	–0.0008	0.082	0.082	0.081

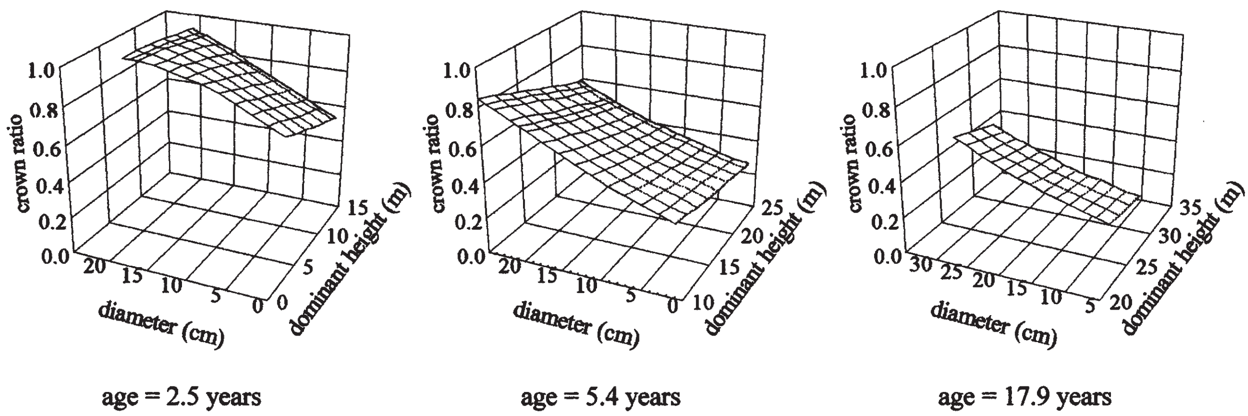


Figure 4. Dynamic of the Richards function for three age classes considered on the evaluation task (and represented by the mean value of each class) and for a density of 1856 ha^{-1}; the range of tree diameter and dominant height in each group was defined according to the observed values.

inverse and the number of live trees per hectare was the best expression of density; an initial tree dimension (diameter) and a measure of stand productivity (dominant height) were also required as explanatory variables.

In this function greater values for age, number of trees or dominant height resulted in smaller crown ratio values, reflecting more advanced stand development stages or greater competitive pressures; in the same stand, at a specific age, an increase in diameter resulted in higher crown ratio values, expressing tree dominance relationships (*figure 4*).

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