

A height-diameter model for *Pinus radiata* D. Don in Galicia (Northwest Spain)

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Abstract – A total of 26 models that estimate the relationship between height and diameter in terms of stand variables (basal area, quadratic mean diameter, maximum diameter, dominant diameter, dominant height, arithmetic mean height, age, number of trees per hectare and site index), were fitted to data corresponding to 9686 trees, using linear and non-linear regression procedures. The precision of the models was then evaluated by cross-validation. The data were collected during two inventories of 182 permanent plots of radiata pine (*Pinus radiata* D. Don) situated throughout Galicia, in the Northwest of Spain. Comparison of the models was carried out by studying the coefficient of determination, bias, mean square error, Akaike's information criterion and by using a F-test to compare predicted and observed values. Best results were obtained with those models that included any independent variable related to the height of the stand (mean or dominant height), although this implies a greater sampling effort for its application. The model of Tomé gave the best height estimates.

Pinus radiata D. Don / forest modelling / Galicia / height-diameter relationship

Résumé – Un modèle hauteur-diamètre pour *Pinus radiata* D. Don en Galice (nord-ouest de l'Espagne). Vingt-six modèles prenant en compte le rapport hauteur-diamètre en fonction des variables de masse (diamètre maximal, diamètre dominant, hauteur dominante, hauteur moyenne arithmétique, âge, nombre d'arbres par hectare, qualité du site, etc.), ont été évalués par rapport aux données relatives à 9686 arbres, grâce à des procédures de régression linéaire et non linéaire. La précision des modèles a ensuite été évaluée par validation croisée. Les données correspondent à deux inventaires réalisés sur 182 parcelles permanentes de *Pinus radiata* D. Don réparties dans toute la Galice, au nord-ouest de l'Espagne. La comparaison des modèles a été effectuée suite à l'étude du coefficient de détermination, du biais, de l'écart quadratique moyen et du critère d'information d'Akaike. Un F-Test a été utilisé pour comparer les valeurs prévues et observées. Les meilleurs résultats ont été obtenus avec les modèles comprenant une variable indépendante en lien avec la hauteur de la masse (moyenne ou hauteur dominante), bien que cela implique un gros effort d'échantillonnage pour son application. Le modèle de Tomé a permis d'obtenir les meilleures évaluations de hauteur.

Pinus radiata D. Don / modélisation de forêt / Galice / rapport hauteur-diamètre

1. INTRODUCTION

Diameter at breast height and total height are the most commonly measured variables in forest inventories. Total height is less frequently used in the development of forest models than diameter, as it is hard and costly to measure, and as a result inaccurate measurements are often made [14]. A single sample of height measurements is therefore usually taken and equations that relate the two variables fitted [26].

These height-diameter relationships are applied to even-aged stands and can be fitted to linear functions, such as second-order polynomial equations [18, 23, 28, 31], or, more usually, to non-linear models [19]. However, the height curves thus obtained for stands do not adapt well to all the possible situations that can be found within a stand. This is for a number of reasons:

1. The height curve of an even-aged stand does not remain constant and is displaced in an increasing direction, for both

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variables, with age [2, 3, 12, 24, 39], i.e. trees that have the same diameter at different times belong to sociologically different classes. The height-diameter curve of a stand is therefore a state function, which is different from the curve of height growth of the stand with age.

2. The height curves for good quality sites will have steeper slopes than those for poor quality sites [34].

3. Clearly, for a particular height, trees that grow in high density stands will have smaller diameters than those growing in less dense stands, because of greater competition among individuals [4, 25, 34, 42].

Therefore, in even-aged stands, in which there is great variation in age, quality and density between cohorts, a single h - d relationship for the whole stand would be the result of many different h - d relationships, with high variability around the regression line. In such cases, to reduce the error involved in estimating heights, the use of a generalised height-diameter equation is recommended, which models the changes in the height-diameter relationship over time [14].

The aim of the present study is to find an equation that can be used to predict the height-diameter relationship in *Pinus radiata* stands in Galicia (north-western Spain) by considering a number of stand variables (dominant diameter, dominant height, age, density, site index, etc.), which may influence the relationship.

2. MATERIALS AND METHODS

2.1. Data used

For this study, total height and diameter at breast height data were used from 355 inventories of a set of 182 permanent plots that the Escuela Politécnica Superior de Lugo (University of Santiago de Compostela) has established in pure, even-aged stands of *Pinus radiata* D. Don throughout Galicia. The plots are square or rectangular, with dimensions varying between 25×25 and 30×40 metres. The number of trees per plot ranges between 30 and 145 depending on the stand density. The plots were installed to give the greatest variety of combinations of age, density and site quality.

Data were collected in two stages; the plots were established and the first inventory carried out between 1995 and 1996, and the second inventory was carried out between 1998 and 1999. Between the two measurements, plots located in thinned stands were remeasured, so that some plots were inventoried three times.

In each plot the diameter at breast height of all of the trees was crosswise measured, using calipers, to the nearest millimetre. Heights were measured using a Blume-Leiss hypsometer in a sample of 30 trees chosen at random from all of the trees in the plot.

The following stand variables were calculated from the data collected in the inventories: basal area, quadratic mean diameter, maximum diameter, dominant diameter and height (using Assman's criterion for both), mean height, age (because the stands are even-aged, the age was calculated from the year of planting), density and site quality, defined as the dominant height (expressed in metres) that the stand reaches at 20 years and determined from the site quality curves available for this specie in Galicia [35].

The mean, maximum and minimum values and standard deviations of the main dendrometric and stand variables are given in *table I* and *table II*, respectively.

Table I. Characteristics of the tree samples used for model fitting.

Variable	Sample for model fitting ($N = 9686$)			
	Mean	Maximum	Minimum	Standard deviation
h	17.29	36.40	3.00	6.19
d	23.90	62.95	5.00	11.93

Table II. Characteristics of the plots from which the samples of trees used for model fitting were taken.

Variable	Sample for model fitting ($N = 9686$)			
	Mean	Maximum	Minimum	Standard deviation
G	53.66	218.50	0.98	39.46
d_g	23.70	53.84	6.04	9.66
D_{\max}	38.82	80.70	11.10	13.85
D_0	33.49	64.75	10.33	11.11
H_0	19.98	34.02	5.80	5.70
H_m	17.13	30.17	3.95	5.29
t	23.28	41.00	5.00	8.50
N	996.11	4864	216.70	550.42
IS	18.97	26.8	13.00	3.24

2.2. Models analysed

A large number of generalised height-diameter equations have been reported in the forestry literature, many of which have been developed for a particular species or specific area. For this study, we have considered the most commonly used, as well as those developed for *Pinus radiata*. Finally, we analysed the 26 generalised height-diameter equations given in *table III*; these are classified according to the real sampling effort in the following groups:

Group 1: Low sampling effort models, including those models which need diameter measurements and knowledge of age in some cases.

Group 2: Medium sampling effort models, including models which need measurements of diameter and of a sample of tree heights.

Group 3: High sampling effort models, including models which need the knowledge or measurements of stand age as well.

The terminology used in the description of the models is as follows:

h = total height of tree, in m; d = diameter at breast height over bark, in cm; G = basal area of the stand, in $\text{m}^2 \text{ha}^{-1}$; d_g = quadratic mean diameter of the stand, in cm; D_{\max} = maximum diameter of the stand, in cm; D_0 = dominant diameter of the stand, in cm; H_0 = dominant height of the stand, in m; H_m = mean height of the stand, in m; t = age of the stand, in years; N = number of trees per hectare; SI = site index, in m; \log = logarithm₁₀; \ln = natural logarithm; b_i = regression coefficients to be determined by model fitting.

The measurement of the mean height requires a greater sampling effort that may allow the limitation of future use of models that include this variable. To avoid this problem, it has been obtained a relationship between the mean height and the dominant height of the stand, the latter value being easier to obtain on field. The resulting equation is the following: $H_m = -1.4497 + 0.9295 \cdot H_0$ ($R^2_{\text{adj}} = 0.9504$; $MSE = 1.4860$).

Table III. Generalized height-diameter models analyzed.

Author(s)	Expression	Group
Curtis [13]	$h = 10^{\left(b_0 + b_1 \frac{1}{d} + b_2 \frac{1}{t} + b_3 \frac{1}{d_g \cdot t}\right)}$	1
Cox (I) [11]	$h = e^{(b_0 + b_1 \cdot \ln d_g + b_2 \cdot \ln N + b_3 \cdot \sqrt{d})}$	1
Clutter and Allison [10]	$h = 1.3 + 10^{\left(b_0 + b_1 \frac{1}{d} + b_2 \frac{1}{\sqrt{t}} + b_3 \frac{1}{d \cdot \sqrt{t}} + b_4 \frac{\log N}{\sqrt{t}}\right)}$	1
Mønness [30]	$h = 1.3 + \left[b_0 \cdot \left(\frac{1}{d} - \frac{1}{D_0} \right) + \left(\frac{1}{H_0 - 1.3} \right)^{\sqrt[3]{3}} \right]^3$	2
Cañadas et al. I [8]	$h = 1.3 + (H_0 - 1.3) \cdot \left(\frac{d}{D_0} \right)^{b_0}$	2
Cañadas et al. II [8]	$h = 1.3 + \frac{d}{\frac{D_0}{H_0 - 1.3} + b_0 \cdot (D_0 - d)}$	2
Cañadas et al. III [8]	$h = 1.3 + (H_0 - 1.3) \cdot \frac{1 - e^{-b_0 \cdot d}}{1 - e^{-b_0 \cdot D_0}}$	2
Cañadas et al. IV [8]	$h = 1.3 + \left[b_0 \cdot \left(\frac{1}{d} - \frac{1}{D_0} \right) + \left(\frac{1}{H_0 - 1.3} \right)^{\sqrt[2]{2}} \right]^2$	2
Gaffrey [15]	$h = 1.3 + (H_0 - 1.3) \cdot e^{b_0 \left(1 - \frac{d}{d_g} \right) + b_1 \left(\frac{1}{d} - \frac{1}{d_g} \right)}$	2
Sloboda et al. [38]	$h = 1.3 + (H_m - 1.3) \cdot e^{b_0 \left(1 - \frac{d}{d_g} \right)} \cdot e^{b_1 \left(\frac{d}{d_g} - \frac{1}{d} \right)}$	2
Harrison et al. [16]	$h = H_0 \cdot \left(1 + b_0 \cdot e^{b_1 \cdot H_0} \right) \cdot \left(1 - e^{-\frac{b_2 \cdot d}{H_0}} \right)$	2
Castedo et al. [9] Mod.*	$h = 1.3 + \frac{b_0 \cdot H_0^{b_1} \cdot (H_0 - 1.3)}{\left(\frac{D_0 - d}{d} \right)^{b_2}}$	2
Pienaar [33] Mod.**	$h = b_0 \cdot H_0 \cdot \left(1 - e^{-\frac{b_1 \cdot d}{d_g}} \right)^{b_2}$	2
Hui and Gadow [21]	$h = 1.3 + b_0 \cdot H_0^{b_1} \cdot d^{b_2 \cdot H_0^{b_3}}$	2
Mirkovich [29]	$h = 1.3 + (b_0 + b_1 \cdot H_0 - b_2 \cdot d_g) \cdot e^{-b_3/d}$	2
Schröder and Álvarez I [37] ***	$h = 1.3 + (b_0 + b_1 \cdot H_0 - b_2 \cdot d_g) \cdot e^{-b_3/\sqrt{d}}$	2
Cox III [11] Mod. ****	$h = H_m \cdot \left[\begin{array}{l} b_0 + b_1 \cdot H_m + b_2 \cdot \frac{H_m}{d_g} + b_3 \cdot d + \\ + b_4 \cdot \frac{N}{d_g \cdot (H_m \cdot d_g)} \\ d \end{array} \right]$	2
Schröder and Álvarez II [37] ***	$h = 1.3 + (b_0 + b_1 \cdot H_0 - b_2 \cdot d_g + b_3 \cdot G) \cdot e^{-b_4/\sqrt{d}}$	2

Cox II [11] Mod. 1 ****	$h = b_0 + b_1 \cdot H_m + b_2 \cdot d_g^{0.95} + b_3 \cdot e^{-0.08 \cdot d} + b_4 \cdot H_m^3 \cdot e^{-0.08 \cdot d} + b_5 \cdot d_g^3 \cdot e^{-0.08 \cdot d}$	2
Cox II [11] Mod. 2 ****	$h = b_0 + b_1 \cdot H_m + b_2 \cdot d_g + b_3 \cdot e^{b_4 \cdot d} + b_5 \cdot H_m^3 \cdot e^{b_4 \cdot d} + b_7 \cdot d_g^{b_6} \cdot e^{b_4 \cdot d}$	2
Tomé [40]	$h = H_0 \cdot e^{\left(b_0 + b_1 \cdot H_0 + b_2 \cdot \frac{N}{1,000} + b_3 \cdot t \right) \left(\frac{1}{d} - \frac{1}{D_0} \right)}$	3
Bennet and Clutter [5]	$h = e^{b_0 + b_1 \cdot IS + b_2 \cdot \frac{N}{100} + b_3 \cdot \frac{1}{t} + b_4 \cdot \frac{1}{d}}$	3
Lenhart [27]	$h = \frac{H_0}{e^{b_0 \cdot \left(\frac{1}{d} - \frac{1}{D_{\max}} \right) \left(b_1 + b_2 \cdot \ln N + b_3 \cdot \frac{1}{t} + b_4 \cdot \ln H_0 \right)}}$	3
Amateis et al. [1]	$h = b_0 \cdot H_0^{b_1} \cdot 10^{\left(\frac{b_2}{t} \cdot \left(\frac{1}{d} - \frac{1}{D_{\max}} \right) \right) \left(b_3 + b_4 \cdot \frac{\log N}{t} \right)}$	3
Burkhardt and Strub [6]	$h = e^{b_0 + b_1 \cdot \ln H_0 + b_2 \cdot \frac{1}{t} + b_3 \cdot \frac{\ln N}{d} + b_4 \cdot \frac{1}{d \cdot t} + b_5 \cdot \frac{1}{d}}$	3
Pascoa [32]	$h = b_0 \cdot H_0^{b_1} \cdot G^{b_2} \cdot N^{b_3} \cdot e^{\frac{b_4 + b_5}{t \cdot d}}$	3

* Generalized height-diameter function obtained relating the parameters of SBB function [22] to stand variables.
 ** Modification of the original model omitting the parameter associated with the number “e” as its value was close to one and the asymptotic standard error was very high.
 *** Modifications of the model of Mirkovich [29] to reduce the bias in the height estimates.
 **** Modification of the original model using the arithmetic mean height of the stand (H_m) as the independent variable instead of the mean value of the maximum and minimum stand heights, which are harder to measure in the field.

Estimations obtained with the above equation (instead of the observed values) were used for the adjustment of the models that include mean height of the stand (H_m) as an independent variable.

2.3. Statistical analysis

Most of the models described above are non-linear, therefore model fitting was carried out with the NLIN procedure of the SAS/STAT™ statistical programme [36] using the Gauss-Newton algorithm [17]. The initial values of the parameters for starting the iterative procedure were obtained, where possible, by previously linearizing the equation and fitting it to the data by ordinary least squares, using the REG procedure of the same statistical programme. When it was not possible to linearize the equation, values obtained by other authors in similar studies were used.

Comparison of the model estimates was based on graphical and numerical analysis of the residuals and values of four statistics: the bias \hat{Z}_r , which evaluates the deviation of the model with respect to the observed values; the mean square error (*MSE*), which analyses the precision of the estimates; the adjusted coefficient of determination (R^2_{adj}), which reflects the part of the total variance that is explained by the model and which takes into account the number of parameters that it is necessary to estimate; and finally, the relative values of Akaike’s information criterion (Δ_i), which is an index for

Table IV. Values of the statistics for fitting and cross-validation for group 1 models.

MODELS	FIT						CROSS - VALIDATION				
	Variables	R^2_{adj}	Bias	MSE	Δ_j	MEF_{adj}	pred. vs. obs. F-Test		Bias	MSE	Δ_j
							F VAL.	Pr > F			
Curtis [13]	d, d_g, t	0.7515	0.0567	9.5314	10727.1099	0.7512	17.56	<0.0001	0.0569	9.5393	10720.3448
Cox I [11]	d, d_g, N	0.7362	-0.0563	10.1168	11304.4445	0.7359	18.71	<0.0001	-0.0565	10.1272	11299.5841
Clutter and Allison [10]	d, t, N	0.7317	0.0382	10.2890	11468.9820	0.7311	6.87	0.001	0.0387	10.3093	11473.2493

selecting the best model, based on minimising the Kullback-Liebler distance [7]. The expressions for these statistics are as follows:

$$\text{Bias: } \bar{E} = \frac{\sum_{i=1}^N (Y_i - \hat{Y}_i)}{N};$$

$$\text{Mean square error: } MSE = \frac{\sum_{i=1}^N (Y_i - \hat{Y}_i)^2}{N - p};$$

$$\text{Coefficient of determination: } R^2 = 1 - \frac{\sum_{i=1}^N (Y_i - \hat{Y}_i)^2}{\sum_{i=1}^N (Y_i - \bar{Y})^2};$$

Adjusted coefficient of determination:

$$R^2_{adj} = 1 - (1 - R^2) \cdot \left(\frac{N - 1}{N - p} \right);$$

Akaike's information criterion differences:

$$\Delta_j = N \cdot \ln \hat{\sigma}_j^2 + 2 \cdot K_j - \min(N \cdot \ln \hat{\sigma}_j^2 + 2 \cdot K_j);$$

where Y_i , \hat{Y}_i and \bar{Y} are the observed, predicted and mean values of heights, respectively; N is the total number of data used in fitting the model; p the number of parameters to estimate; R^2 the coefficient of determination; K_j the number of parameters in model j plus 1 ($K_j = p + 1$) and $\hat{\sigma}^2$ an estimate of the error variance of model j , calculated

$$\text{as: } \hat{\sigma}^2 = \frac{\sum_{i=1}^N (Y_i - \hat{Y}_i)^2}{N}.$$

To analyse the predictive capacity of the equations a cross-validation was carried out. The values of the prediction residuals obtained in the cross-validation were used to calculate the bias, mean square error (MSE), Akaike's information criterion differences (Δ_j) and the precision of the model (MEF_{adj}):

$$MEF_{adj}^2 = 1 - \frac{\sum_{i=1}^{N'} (Z_i - \hat{Z}_i)^2}{\sum_{i=1}^{N'} (Z_i - \bar{Z})^2} \cdot \left(\frac{N' - 1}{N' - p} \right)$$

where Z_i and \bar{Z} are the observed and mean values of heights, respectively, \hat{Z}_i is the prediction residual; N' is the total number of data used and p is the number of parameters to be estimated.

To evaluate the possible existence of bias, the linear model $Z_i = a \cdot \hat{Z}_i + b$ ([20, 41]) was fitted, and a F-test was used to check the null hypothesis (the absence of bias) i.e. that the slope of the straight line was equal to 1 at the same time as the independent term (b) was equal to 0.

3. RESULTS

3.1. Model fitting phase

In order to be able to interpret and compare the results more easily and because of the large number of equations analysed, the models were classified in three groups according to the sampling effort, as pointed out in the Materials and Methods section above.

Tables IV, V and VI show the values of the statistics used to compare the models in the fitting phase and in the cross-validation for the groups 1, 2 and 3 respectively.

The results of fitting and cross-validation for the models of group 1 are the poorest, as could be expected. The use of independent variables related only with either the diametrical distribution or with the age of the stand does not appear to be sufficient explanation for the variability observed on height values. Therefore, it is advisable the inclusion of an additional variable in order to improve the estimates.

Finally, it's important to emphasise the poor behaviour of the model of Clutter and Allison [10], in spite of this model was used to estimate the height of individual trees in a growth model for *Pinus radiata* in New Zealand.

The values of the statistics of the models included in the group 2 (table V) show that the second modification of the Cox [11] model (Cox II, Mod. 2) is the equation of this group that most accurately estimates the height. This equation improves the accuracy of the first modification (Cox II, Mod. 1) due to the lack of restrictions for the values of the exponent of the independent variables. Only the exponent affecting d_g has been restricted, due to a lack of convergence. The good results obtained with this model are consistent with those obtained by Cox [11] for the same species (*Pinus radiata*) in Chile.

The models of Mønness [30] and Cañadas et al. IV [8] also fit well to the data. The advantage of these models is that they are functions of one single parameter, although the bias and MSE were slightly higher than those of the modified versions of the model of Cox II [11].

Table V. Values of the statistics for fitting and cross-validation for group 2 models.

MODELS	FIT					CROSS - VALIDATION					
	Variables	R^2_{adj}	Bias	MSE	Δ_i	MEF _{adj}	pred. vs. obs. F-Test		Bias	MSE	Δ_i
							F VAL.	Pr > F			
Mønness [30]	d, D_0, H_0	0.9130	0.0399	3.3371	558.7367	0.9129	42.29	< 0.0001	0.0399	3.3382	547.1393
Cañadas et al. I [8]	d, D_0, H_0	0.9093	0.0909	3.4771	956.7995	0.9093	142.46	< 0.0001	0.0908	3.4782	946.1194
Cañadas et al. II [8]	d, D_0, H_0	0.8895	0.5730	4.2357	2869.4106	0.8904	748.95	< 0.0001	0.5728	4.2391	2862.2331
Cañadas et al. III [8]	d, D_0, H_0	0.9090	0.1743	3.4893	991.6849	0.9090	167.04	< 0.0001	0.1742	3.4908	981.0705
Cañadas et al. IV [8]	d, D_0, H_0	0.9145	0.0361	3.2760	379.8576	0.9145	6.06	0.0023	0.0361	3.2771	368.2144
Gaffrey [15]	d, d_g, H_0	0.7772	-2.1953	8.5411	9662.6009	0.8020	6666.1	< 0.0001	-2.1957	8.5448	9651.9901
Sloboda et al. [38]	d, d_g, H_m	0.8765	-0.0258	4.7334	3945.4715	0.8764	1.46	0.2328	-0.0259	4.7374	3940.9090
Harrison et al. [16]	d, H_0	0.8992	-0.0307	3.8654	1987.2401	0.8991	10.46	< 0.0001	-0.0307	3.8681	1979.2874
Castedo et al. [9] Mod.	d, H_0	0.8575	-0.0336	5.4655	5339.3047	0.8573	8.9	0.0001	-0.0339	5.4720	5336.1777
Pienaar [33] Mod.	d, d_g, H_0	0.9039	-0.0960	3.6846	1520.2139	0.9038	132.39	< 0.0001	-0.0960	3.6873	1512.5555
Hui and Gadow [21]	d, H_0	0.8858	-0.0160	4.3818	3203.8350	0.8856	2.41	0.0896	-0.0162	4.3860	3198.4347
Mirkovich [29]	d, d_g, H_0	0.9055	0.0273	3.6246	1362.2359	0.9054	8.18	0.0003	0.0273	3.6283	1357.3113
Schröder and Álvarez I, [37]	d, d_g, H_0	0.9106	-0.0041	3.4291	825.0991	0.9105	0.19	0.8285	-0.0041	3.4325	820.0672
Cox III [11] Mod.	d, d_g, H_m, N	0.8763	-0.0032	4.7428	3967.6250	0.8761	0.1	0.9085	-0.0036	4.7503	3973.1747
Schröder and Álvarez II [37]	d, G, d_g, H_0	0.9106	-0.0041	3.4292	826.3648	0.9104	0.19	0.8271	-0.0042	3.4335	823.9297
Cox II [11] Mod. 1	d, d_g, H_m	0.9135	0.0000	3.3168	504.6334	0.9133	0	0.9993	0.0002	3.3232	508.4968
Cox II [11] Mod. 2	d, d_g, H_m	0.9156	0.0000	3.2367	270.9605	0.9153	0.00	0.9986	0.0001	3.2453	281.8632

Table VI. Values of the statistics for fitting and cross-validation for group 3 models.

MODELS	FIT					CROSS - VALIDATION					
	Variables	R^2_{adj}	Bias	MSE	Δ_i	MEF _{adj}	pred. vs. obs. F-Test		Bias	MSE	Δ_i
							F VAL.	Pr > F			
Tomé [40]	d, D_0, H_0, t, N	0.9179	-0.0028	3.1491	0.0000	0.9178	0.21	0.812	-0.0029	3.1526	0.0000
Bennet and Clutter [5]	d, t, N, IS	0.8574	0.0379	5.4701	5349.6127	0.8568	12.8	< 0.0001	0.0382	5.4896	5369.1342
Lenhart [27]	d, H_0, t, N, D_{max}	0.9118	0.0221	3.3821	697.4526	0.9117	6.61	0.0013	0.0221	3.3865	695.4285
Amateis et al. [1]	d, H_0, t, N, D_{max}	0.9113	0.0107	3.4028	751.6177	0.9110	1.63	0.196	0.0105	3.4108	759.5348
Burkhart and Strub [6]	$d, H_0, t, N,$	0.9065	0.0152	3.5857	1259.8154	0.9063	3.02	0.049	0.0150	3.5914	1260.2069
Pascoa [32]	d, H_0, t, G	0.9005	0.0399	3.8157	1861.7774	0.9003	19.03	< 0.0001	0.0397	3.8220	1863.1169

The six models classified in group 3 (table VI) have similar results to those of group 2, although the model of Tomé [40] gives the best performance of all the models tested, according to the values of the statistics used to compare the models in the fitting phase and the cross-validation. The results were slightly better than those obtained with the model of Cox II [11] when the values of the exponents of the independent variables were not restricted.

Plot of residuals versus the heights predicted in the fitting phase of the model of Tomé [40] are shown in figure 1. There

was no reason to reject the hypotheses of normality, homogeneity of variance and independence of residuals.

Plot of the observed heights versus the predicted heights in the cross-validation of this model are shown in figure 2. The criterion to evaluate the behaviour of the model was the determination coefficient of the straight line fitted between the observed and predicted heights. The chart shows no tendency toward the overestimation or underestimation of height values.

To analyse the behaviour of the two best models (Tomé [40] and Cox II [11] Mod. 2) the values of the bias and the

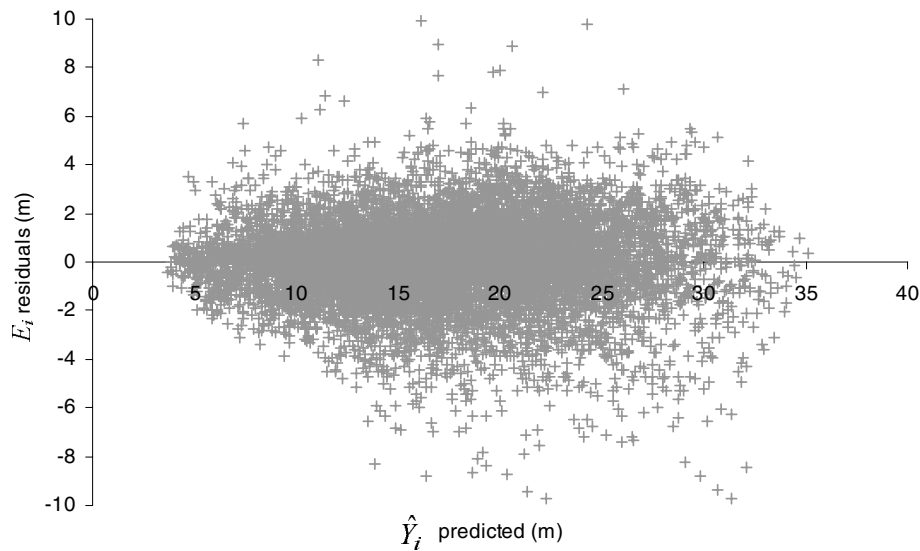


Figure 1. Plot of residuals versus predicted values in the fitting phase for the model of Tomé [40].

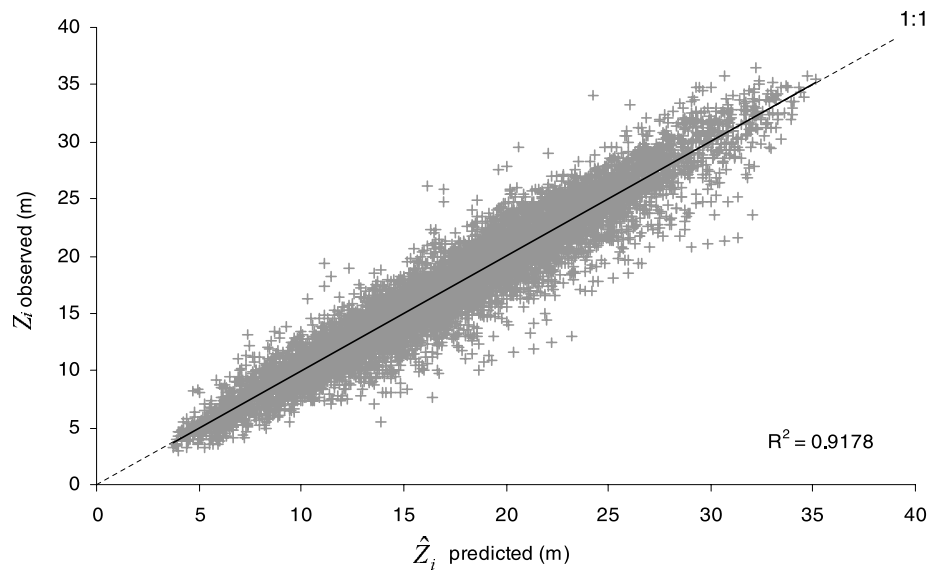


Figure 2. Plot of observed values versus predicted values in the cross-validation for model of Tomé [40]. The solid line represents the linear model fitted to the scatter plot of data. The dotted line represents the diagonal.

mean square error were calculated and plotted against diameter classes, for the fitting phase and the cross-validation. (figures 3 and 4).

Despite a slight trend to overestimation for the higher diameter classes, the model proposed by Tomé [40], which includes five independent variables, gives a better performance.

The good performance of this model may be due, in part, to the inclusion of the stand age, which is an important variable

in the consideration of even-aged, uniform stands [8] (which the stands in plantations usually are) because, in these cases, the age gives an indication of the mean size of the individual trees in the stand.

In general, the inclusion of new independent variables in the height-diameter equation reduced bias and increased the precision of the model. However, the increase in accuracy of the estimations is usually associated with a larger sampling

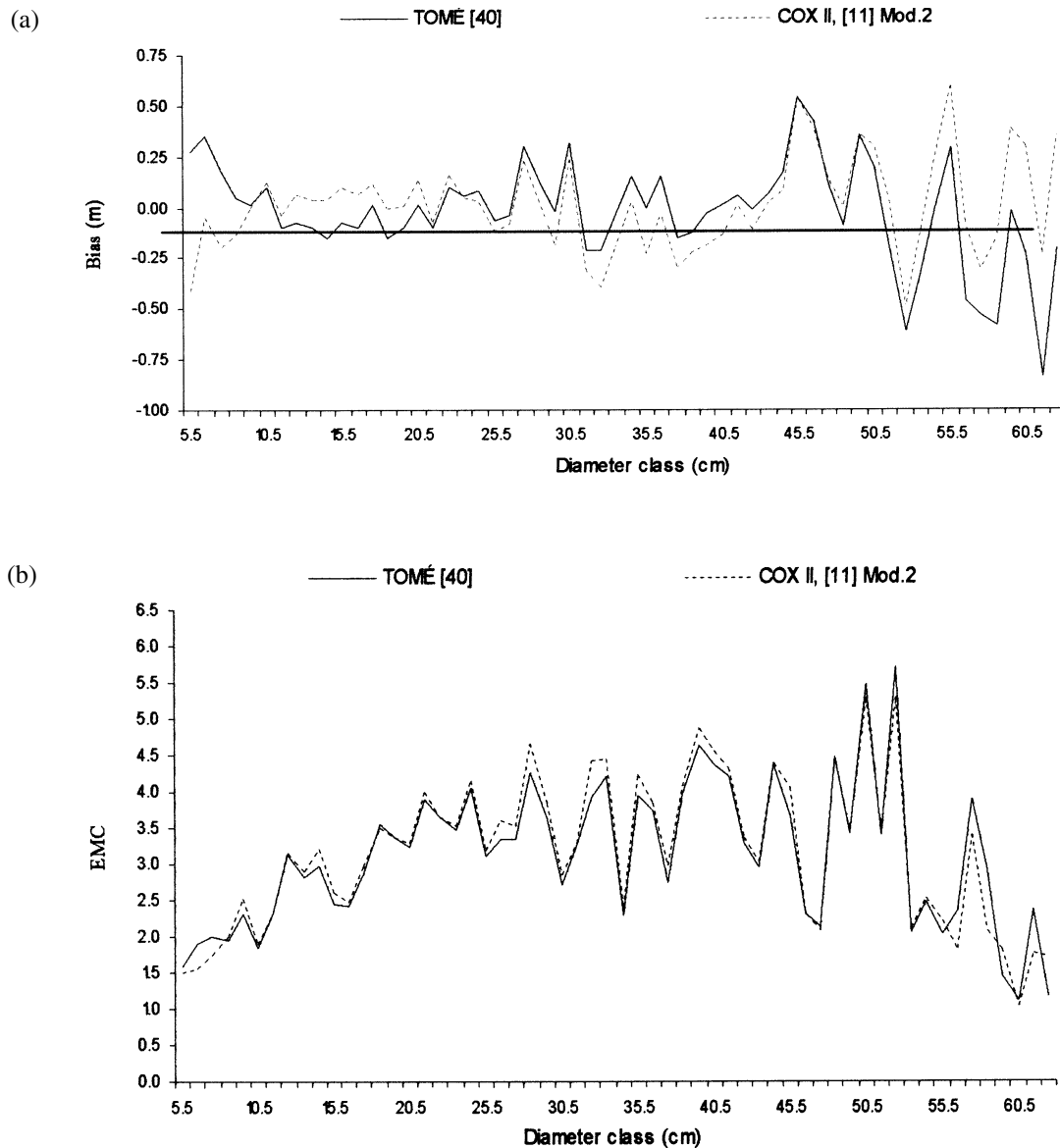


Figure 3. Values of (a) bias and (b) mean square error obtained for diameter classes in the fitting phase of the two best models (Tomé [40] and Cox II [11] Mod. 2).

effort due to the greater number of independent variables that must be measured in the field. The model of Tomé [40] could offer a balance between the accuracy of the model and the sampling effort, because the value of age is well-known if the date of plantation is available.

4. CONCLUSIONS

The inclusion of the mean height or of the dominant height as an independent variable in the height-diameter equations appears to be necessary in order to achieve acceptable

predictions. This requires the measurement of at least one sample of heights for the practical application of the equation.

The best predictions of height were obtained by the model of Tomé [40], which uses diameter (d), dominant diameter (D_0), dominant height (H_0), age (t) and number of trees per hectare (N) as independent variables; this was followed in performance by the modified version 2 of the model of Cox II [11], which depends on three variables (d , d_g , H_m).

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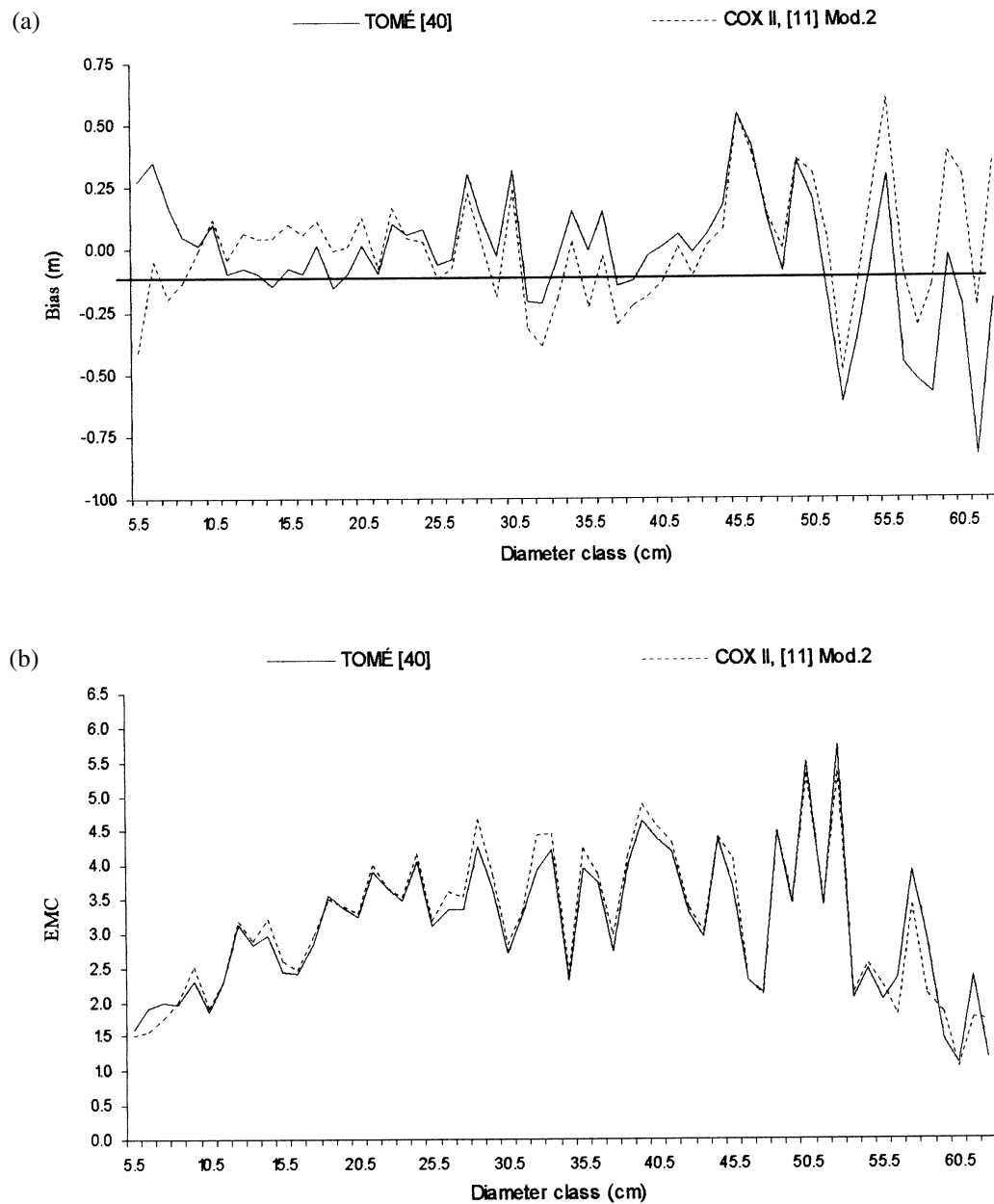


Figure 4. Values of (a) bias and (b) mean square error obtained for diameter classes in the cross-validation of the two best models (Tomé [40] and Cox II [11] Mod. 2).

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