

A flexible radial increment model for individual trees in pure even-aged stands

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Abstract – To capture the common feature of various individual growth models for trees in pure and even-aged stands, a generic model with a form parameter according to each species is proposed. With a final objective of implementing this model into simulation software of silvicultural treatments, particular care was taken to structure the model for consistent behaviour outside the data fields of calibration and to limit the independent variables to those available to managers. The first analysis, using data from a young spruce stand, allowed introduction of simple variables (basal area and dominant height) to express competition effects. Then the model was fitted on a large data set to take into account silvicultural treatments and fertility conditions. Finally the model tested on spruce was directly adjusted for Douglas-fir and was found to be sufficiently flexible to describe growth of this species. This newly derived model is a relatively generic tool, which can be fitted for other species in regular stands.

tree growth model / regular stand / silviculture / software / simulation / spruce / Douglas-fir

Résumé – Un modèle générique de croissance radiale d'arbre en peuplements purs et réguliers. Un modèle générique de croissance de type arbre, pour des peuplements purs et réguliers, est proposé en utilisant les points communs de différents modèles de la littérature et en tenant compte à travers un paramètre des différentes formes observées selon les essences. L'utilisation de ce modèle étant une implémentation informatique dans un simulateur de sylviculture, sa forme a été contrainte pour un comportement robuste en dehors de la zone de validité, tandis que les variables directrices ont été choisies pour faciliter l'utilisation par des gestionnaires forestiers. La première étape basée sur un jeune peuplement clinal d'épicéa a permis d'introduire simplement l'effet de la compétition à l'aide de deux variables explicatives (surface terrière et hauteur dominante). L'utilisation d'un large réseau d'essais a ensuite permis de prendre en compte l'effet simultané de la sylviculture et de la fertilité. Enfin ce modèle a directement été ajusté sur une base de données de croissance de Douglas et s'est parfaitement adapté à une forme de croissance plus progressive.

modèle de croissance d'arbre / peuplement régulier / sylviculture / logiciel / simulation / épicéa / Douglas

1. INTRODUCTION

Growth models are increasingly introduced into simulation software of silvicultural treatments. These models have been for a long time “tools of experts”, built and used by researchers or experts. Now, they are often put together end to end into software and then used by forest managers, who did not take part in the design.

This new aim of the models has been underestimated: computerized models could be used out of validity area without any critical analysis on results. In this context it appears essential to work on the structure and form of these models to ensure that they exhibit a consistent behavior in simulation software.

Firstly to account for validity limits of models fitted on reduced data sets, validity checking could be introduced inside software

to stop results outside of the validity range. However models are often a function of many variables, thus the validity range may be difficult to describe and implicit model extrapolations (inside space limits, but not covered by data set) may result.

Improving model form and structure is an alternative solution to ensure coherent behavior outside the validity range (structured and constrained models are favored over completely empirical models like multiple regressions).

In addition certain models also use independent variables that can be difficult to obtain except from data resulting from experimental tests (for instance information from the origin stand). These explanatory variables are often kept because “R-square” is improved during model building but they are difficult to estimate for managers.

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Since 1996, AFOCEL has developed simulation software of silvicultural treatments (OASIS [13]) for 3 major species (Norway spruce, Maritime pine and Douglas-fir) in pure even-aged stands. In this project, a particular emphasis is made to ensure robust behavior of models by external users such as forest managers. This paper deals with new results concerning the function of individual tree growth in circumference at breast height.

This approach on the constrained model form with few explanatory variables consists moreover in developing relatively generic tools that can be fitted for other species in regular stands. The first stage of construction on Norway spruce is presented here, along with its adaptation to Douglas-fir.

2. STATE OF THE ART

Houllier et al. [10], then Gourlet-Fleury [9] present a review of different growth models. To describe one stand, scale goes from distance dependent tree growth model to general stand model, without reference to individual trees. OASIS is a distance non-dependent tree growth model that allows interactive relationships between silvicultural treatments and simulated growth reactions. In addition the growth pattern of each tree can be recalled, which is essential information to characterize the internal quality of individual stems. On the other hand exact location of trees is not required in these models.

Tree growth in these models can be described with one equation by tree (example of Zhang et al. [17] to describe growth of young pines), with additional constraint to take into account stand saturation (potential-reducer model [3, 16]).

Other models are focused on growth relation between trees during the growth period. This approach applies the very strong social ordered structuring between trees that is established in regular stand after canopy closure [6]. For instance, Alder [1] used this structuring by describing growth with a one-to-one relationship between individual growth and initial relative size.

Dhôte [7, 8] proposed a segmented linear model between individual basal area increment and initial circumference at breast height (initially for beech, then for oak). It has an initial part of null growth for understorey trees and an increasing growth part for main vegetation storey trees. Pain [12, 14] and Najjar [11] used this same form respectively for Norway spruce and for Maritime pine in pure even-aged stands (without the null growth for Maritime pine for trees suppressed sufficiently to lead to mortality). Finally Bailly et al. [2] used a segmented cubic model for Douglas-fir, to take into account a more gradual increase of increment for transitional trees, between suppressed and dominant.

The general form of Dhôte's model is quite interesting since a threshold of social tree status (limit suppressed/dominant) and a potential global growth (slope increment increase for dominant trees) are introduced at the same time. Variations around this model simply aim at making the segmented form slightly more flexible (in particular for Douglas-fir), i.e. a more or less fast increment increase around the threshold for transitional trees. This is why a nonlinear hyperbolic model is proposed, having two segments of Dhôte's model for limits.

To take into account a model evolving with age, environment or competition, parameters are then related to independent stand variables. These relationships are usually multiple regres-

sions of different stand variables (stand basal area, number of trees per hectare, dominant or mean height, dominant or mean diameter or basal area, basal area weighted mean diameter or height, site index, crown ratio, different competition index, dendrometric data for dominant trees, past mean growth, etc.), quite contrasted from one model to another one [2, 12, 15]. Simple relationships with few explanatory variables, available to managers, are proposed here to be fitted to different species.

3. METHODOLOGY

A general model describes tree basal area increment (Ig) as a function of initial circumference C_i , with 3 parameters: threshold A , slope P , and form parameter m : m is greater than 1 ($m = 1$ for Dhôte's model).

The hyperbole equation is given by 3 additional constraints:

Model passes point (0,0) i.e. basal area increments (Ig) are null for initial null circumference (C_i);

Towards lower C_i (limit $-\infty$), model tends towards $Ig = 0$. To ensure a passage through (0,0), asymptote is slightly negative: $Ig \approx A(1 - m)$;

Towards stronger C_i (limit $+\infty$), model tends towards a linear $Ig \approx P(C_i - A)$.

Corresponded equation is then (Fig. 1):

$$Ig = P \left(\frac{C_i - mA + \sqrt{(mA + C_i)^2 - 4AC_i}}{2} \right). \quad (1)$$

To take into account heteroscedasticity of data, observations were weighted by $1/C_i^2$.

This model was studied for Norway spruce, using a data set especially collected for this project. These data came from experimental plots with continuous gradient of density. Analysis of these data made it possible: (i) to test model for annual data, (ii) to better describe competition relationships in juvenile stage, (iii) and to choose some simple explanatory variables, easily available to managers.

This model was then fitted on the database from AFOCEL's network of experimental trials (stand densities between 350 and 4 500 trees per ha (at the first thinning); site index (dominant height at 50 years) between 20 and 35 m).

This model was finally tested on Douglas-fir to study its generic capability.

It was then implemented in interactive OASIS software to simulate various forestry scenarios (in pure and even-aged stands for different thinning treatments).

4. DATA

4.1. AFOCEL's trials

The main database used came from AFOCEL's network of experimental permanent plots, which has received periodic measurements since 1968. Available information is presented in Table I for Norway spruce and Douglas-fir. For each individual tree there corresponds complete stand information (age,

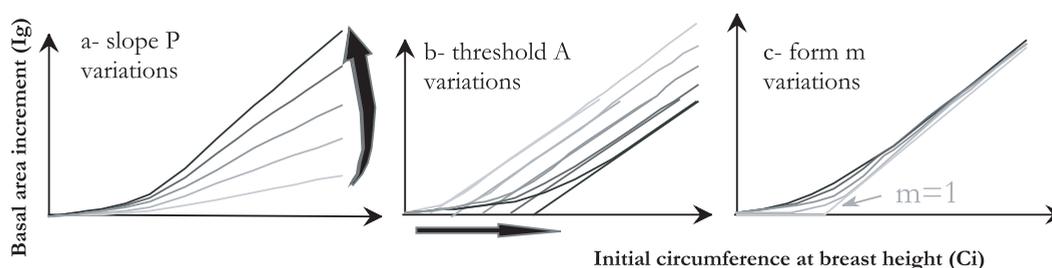


Figure 1. Sensitivity analysis for individual tree growth model: variations of one of the parameters, others being constant ($P = 10$; $A = 300$; $m = 1.05$). a: increase in P ; b: increase in A ; c: increase in m .

Table I. Database characteristics for Norway spruce and Douglas-fir from the AFOCEL's network.

	Principal data base	
	Norway spruce	Douglas-fir
Trials number	19	21
Plots number	40	37
Nb of stand increments	341	465
Individual tree increments	33 277	32 107

Table II. Density classes and number of sampled trees.

Classes of density	Densities	Tree nb in 1990
N1	< 1500 st/ha	60
N2	1 500–2 000 st/ha	40
N3	2 000–2 500 st/ha	40
N4	2 500–3 000 st/ha	42
N5	3 000–4 000 st/ha	42
N6	> 4 000 st/ha	38

initial density, actual density, total basal area G , dominant height H_0 , etc.).

4.2. Vercel's trial

Additional measurements were made on a large sample of trees, coming from an experimental trial installed in 1977 with a continuous gradient of spacing in two perpendicular directions, representing densities from 730 to 8 264 stems/ha.

Thirty-eight trees were already sampled in 1993 in 3 repeated plots. In 1999, 224 other trees were sampled in these plots and one complete fourth plot. Trees were selected far from previous sampling, so that around 50 trees were selected in the first 3 plots and 75 in the fourth one.

The height was regularly measured on all trees from plantation until the present, which enabled us to make very precise measurements of height growth by readjusting units of first years growth to these previous measurements, and by finding all growth unit limits (bud scale scars on the bole) until 1999.

Discs at breast height were also taken to perform ring analysis and to measure annual growth according to 4 fixed radii (north, south, east and west).

To be able to increase the number of trees by density, we built 6 density classes (Tab. II). Stand information was then computed (G , H_0 , N). For these groups, a local basal area G was calculated (sum of individual basal areas/sum of space available for each tree). In agreement with Deleuze et al. [5] these data showed that dominant height H_0 was not affected by density. Hence all densities were pooled together when computing dominant height for each plot.

5. RESULTS

5.1. Vercel

For each year, a global model (for all densities) is compared (through a F-test) to a model with one parameter by density (the slope P , the threshold A or the form m : see Tab. III). Residuals (of the global model) are compared between densities (Fig. 2). Four stages are then characterized:

Stage 1: 1981–1984: no density effect is observed (the global model is retained). Stand is still very young and canopy is not closed.

Stage 2: 1985–1990: residuals decrease gradually with density (weaker growth in denser part of stand). Canopy is closed and competition is stronger, which gradually starts in 1985 from highest densities. Effect is mainly sensitive on slope P which decreases with density (model with local slope P by density is retained).

Stage 3: 1991–1994: differences between residuals gradually decrease for highest densities, indicating that there is no more difference between these densities. Density effect is then felt only by the lowest densities. It affects mainly threshold A which increases with density (model with local threshold A is retained).

Stage 4: 1995–1999: no more effect is visible on the model (the global model is retained).

This model shows a gradual density effect on growth relationship. In addition, even when no effect is detected on a model, there is a shift between tree distributions (more small trees in the higher densities result in smaller increments).

With an increase in density, the slope decreases, while the threshold increases, both leading simultaneously to a growth

Evolution of density effect with age

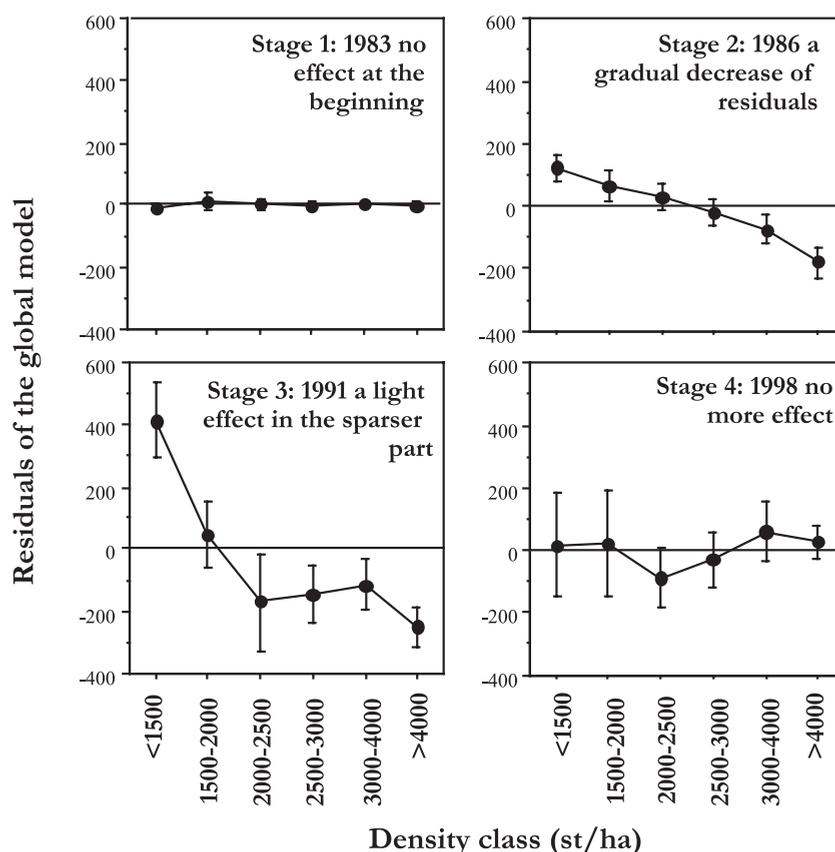


Figure 2. Global model residuals average (without density effect), for each density classes, and with confidence intervals. 4 years examples, others being detailed in text.

decrease (with equal initial sizes, trees grow slowly at higher density). With threshold increase, some co-dominant trees in sparse stands are regarded as suppressed in higher densities.

To take into account density effect (or competition) in a stand, and to scale this effect with age or height, the ratio “total basal area increment/dominant height increment” is used, that increases towards an optimum, like saturation point of total basal area increment for dominant height (Fig. 3).

To describe the exponential form of this saturation, the S function is built:

$$S = e^{-\alpha G/Ho} \quad (2)$$

where α is parameter, G is stand basal area, and Ho dominant height.

For a stand, this function measures the distance from saturation, which indicates deceleration of growth. α is set to 80, to have the same threshold than Figure 3.

To describe this decrease of growth, two relationships are introduced:

$$P = Po \times (1 + p_s \times S) \quad (3)$$

$$A = Ao \times (1 + a_s \times S)$$

where P and A are parameters of equation (1), S is previous saturation function (Eq. (2)), Po , Ao , p_s , and a_s are parameters.

Then the model is simultaneously fitted on all years with 2 global parameters p_s and a_s , taking into account density effect and annual parameters for each year (Po and Ao). This model is retained. Density effect is positive and stronger on slope ($p_s = 1.40 \pm 0.20$), negative and weaker on threshold ($a_s = -0.51 \pm 0.22$).

Compared to a local model (P and A per year and density), this global model describes well the data, but added constraints help to stabilize estimates. Graphically the evolution of density effect is shown to become very weak in 1999 (Fig. 4).

These detailed annual data from Vercel allow one to study separately effects of density or age. Comparatively to Dhôte’s model on beech, density effect does not affect solely slope: initially effect is especially on slope, and then threshold is also affected. Global model however makes it possible to show that density effect is stronger on slope than on threshold.

For this first model, Ao seems to increase with age, and Po is more stable (Fig. 5), as observed by Dhôte [7]. Increase of parameter Ao with a small delay looks like crown recession and could reflect beginning competition (canopy closure).

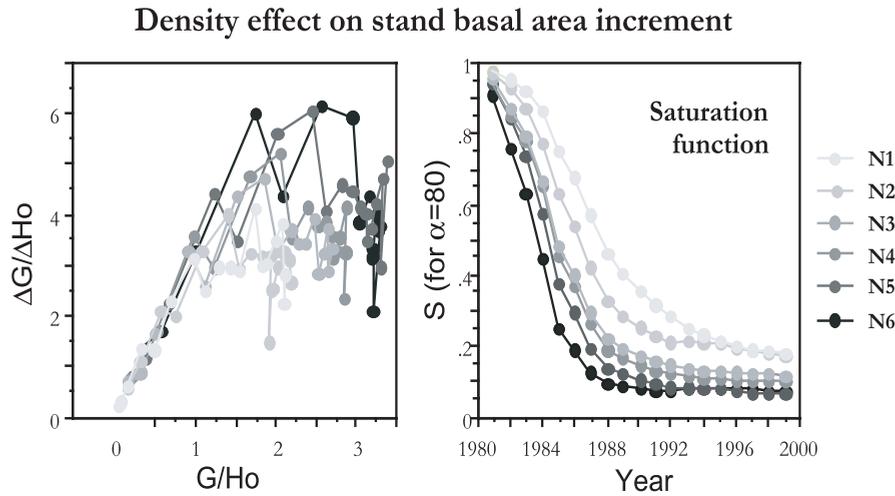


Figure 3. Density effect on basal area increment and associated saturation function *S* for stand “potential” (according the so called “Eichhorn rule”).

Table III. Results for the 4 models fitting (ggg: global model whatever density; lgg: slope *P* depends on density; glg: threshold *A* depends on density; ggl: form *m* depends on density). SSE: Sum of Square Errors. A F-test for the “best” local model (SSE in bold) gives retained model, and its associated *P*-value.

Year	n	SSEggg	SSElgg	SSEglg	SSEggl	Retained model	Associated <i>P</i> -value
1981	196	349.63	337.78	354.41	388.96	ggg	25.79
1982	237	860.73	830.55	768.52	1 227.85	glg	0.01
1983	256	455.51	451.84	451.38	526.05	ggg	81.03
1984	257	662.24	654.17	647.14	808.10	ggg	32.85
1985	260	946.51	867.06	876.08	1 029.35	lgg	0.05
1986	261	357.85	233.90	234.85	391.37	lgg or glg	0.00
1987	262	1 023.24	587.03	600.14	1 095.09	lgg	0.00
1988	262	1 233.18	499.73	496.25	1 418.12	glg or lgg	0.00
1989	262	549.47	239.81	246.42	724.38	lgg	0.00
1990	262	659.08	314.91	328.59	868.79	lgg	0.00
1991	262	488.38	302.61	300.60	754.70	glg	0.00
1992	262	374.14	283.24	276.24	680.97	glg	0.00
1993	224	318.82	258.32	255.33	624.23	glg	0.00
1994	224	312.94	275.25	275.47	540.23	lgg or glg	0.00
1995	224	281.15	261.65	261.22	518.56	ggg	0.69
1996	224	296.93	277.50	275.50	623.85	ggg	0.60
1997	224	286.33	279.70	280.01	717.46	ggg	40.47
1998	221	107.91	104.84	104.51	10 837.53	ggg	22.95
1999	220	221.99	208.39	210.79	439.70	ggg	1.91

5.2. AFOCEL’s network for Norway spruce

When increment period does not exceed one year, a sensitivity analysis shows that 2 to 4 iterations of annual model give comparable results than multiplicative model with $2 \times P$ to $4 \times P$. To take this into account the next model is directly used:

$$I_g = P \times period \times \left(\frac{C_i - mA + \sqrt{(mA + C_i)^2 - 4AC_i}}{2} \right) \quad (4)$$

where *P*, *A* and *m* are parameters of equation (1), *C* is initial circumference and “period” is period of growth.

Parameter *P* is thus automatically brought back to annual increase and then is comparable between each increment, whatever their length of growth.

For each trial, each plot (silvicultural treatment), and each growth period, a local model is fitted, and parameters *A*, *P* and *m* are analyzed according to available explanatory data. A first

Evolution of density effect with age

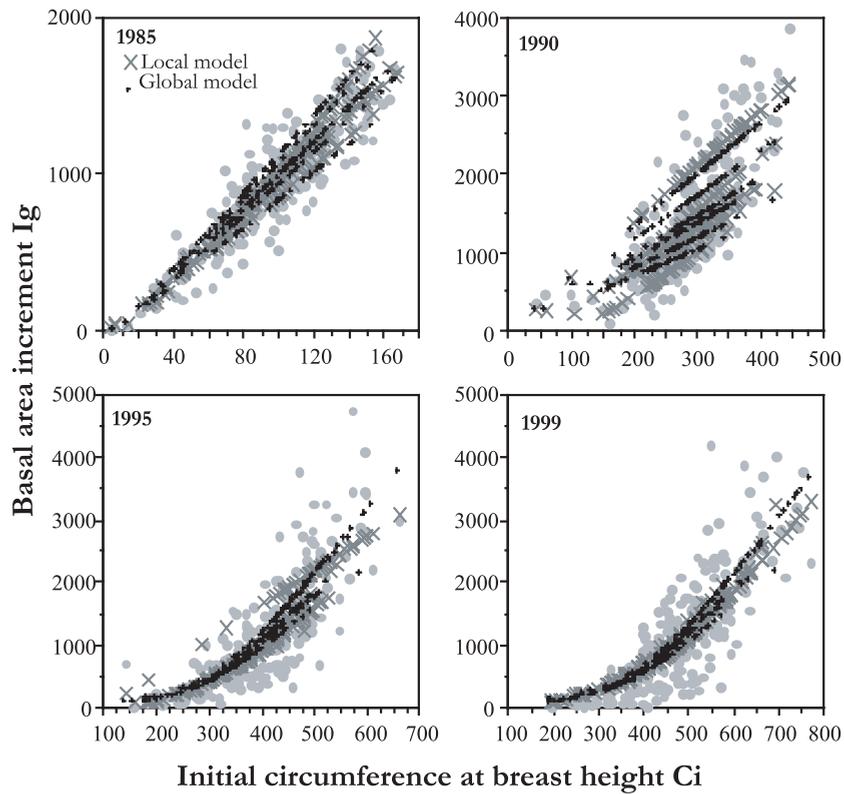


Figure 4. Basal area increment I_g (mm^2) vs initial circumference at breast height (C_i , mm). Fitting example of global model (black +) compared to local model by density (gray x) by way of comparison.

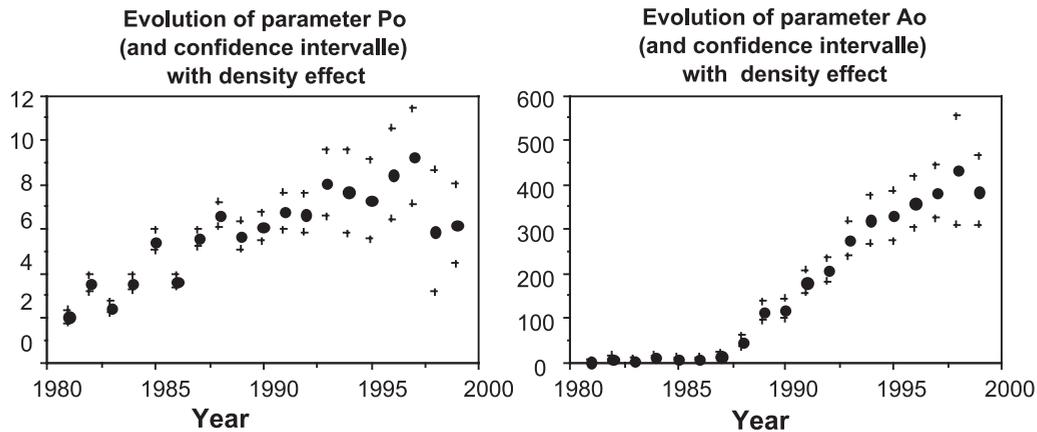


Figure 5. Evolution with age of annual parameters P_o and A_o for global model fitting on Vercel's data (with density effect).

effect on growth is the growth period (stand age and dominant height), and then silvicultural treatment (density and stand basal area).

For each trial, a relationship is found between P and dominant height increment ΔH_o (Fig. 6). Parameter A depends mostly on

age or total dominant height (which is in agreement with Dhôte's model for beech and oak). Form parameter m is quite stable, regardless of stand, plot or period. Age has a very slight increasing effect on form during the young age, but it is quickly stabilized towards a form value close to 1.02. Moreover a weak

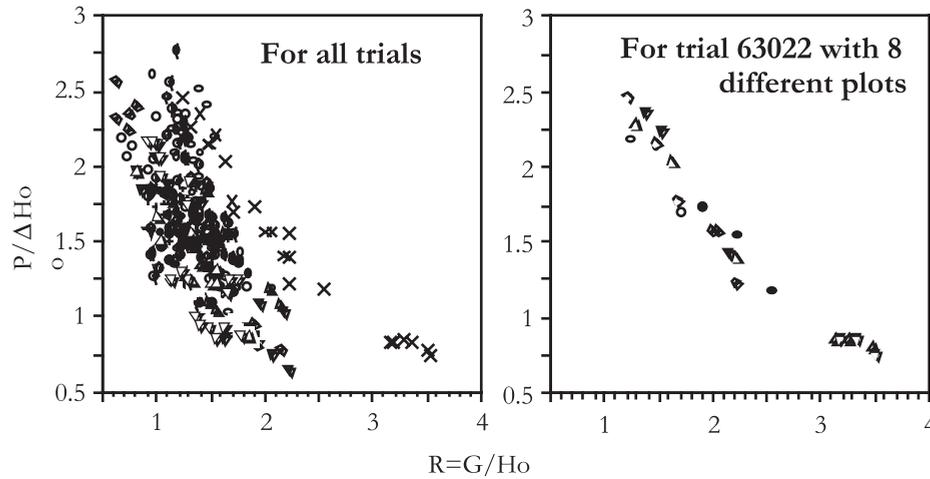


Figure 6. Relationship between slope P and dominant height increment during increment period.

Table IV. Parameters of the 3 final global models for Norway spruce and Douglas-fir, when dominant height is measured or provided by a model (estimated parameters and their standard errors in italics).

Parameters	N. spruce (measured Ho)		N. spruce (estimated Ho)		Douglas-fir (measured Ho)	
Pa	0.2656	<i>0.0108</i>	0.2138	<i>0.0124</i>	0.1852	<i>0.0032</i>
Pb	0.5185	<i>0.0201</i>	0.6246	<i>0.0338</i>	0.0098	<i>0.0032</i>
Pc	1.198	<i>0.1253</i>	0.8567	<i>0.1433</i>	7.3383	<i>0.0700</i>
Aa	-8.7761	<i>0.5181</i>	-15.5394	<i>0.7863</i>	-18.2098	<i>0.1481</i>
Ab	3.2684	<i>0.0895</i>	3.6075	<i>0.1339</i>	5.572	<i>0.0225</i>
Ac	-1.0366	<i>0.03356</i>	-1.0215	<i>0.0314</i>	-0.6734	<i>0.0051</i>
a	0.7873	<i>0.0358</i>	0.649	<i>0.0382</i>	0.2722	<i>0.0038</i>
m	1.0165	<i>0.0019</i>	1.0255	<i>0.0022</i>	1.0312	<i>0.0014</i>

influence of the form parameter on fitting results has been shown by sensitivity analysis. According to these results, a global model is built:

$$A = (Aa + Ab \times Ho) \times (1 + Ac \times \exp(-\alpha \times G/Ho)) \quad (5)$$

$$P = (Pa + Pb \times \Delta Ho) \times (1 + Pc \times \exp(-\alpha \times G/Ho))$$

where α , Aa, Ab, Ac, Pa, Pb, Pc are parameters, G is stand basal area, and Ho dominant height.

This global model with 8 parameters is then fitted on data (Tab. IV) and 88.24% of variance is explained (standard error is 0.8257 C_i cm).

A same model is fitted when measured Ho is replaced by global evaluation of dominant height with a stand growth model (dominant height in simulation software is given by height growth model, then direct fitting is powerful to avoid cumulating errors). Then 86.51% of variance is explained and residual error is 0.8846 C_i cm (see Tab. IV and Fig. 7 for example, trial 54001B). This error is slightly higher because real measured heights bring information on observed variability, even if these measurements are not directly available in simulation software.

As for Vercel, density effect is opposed for the two main parameters: parameter P of slope (Pc > 0) decreases, whereas parameter A of threshold (Ac < 0) increases, leading both to growth reduction with density.

Influence of silvicultural treatments is confirmed on these two parameters. Unlike results obtained by Dhôte for beech, the threshold is also influenced by silviculture. Once more, the ratio G/Ho seems efficient to describe competition pressure on individual tree growth. However parameter α can be estimated and is relatively far from the fixed value for Vercel. Density effect is thus more regular, even for values approaching maximum values of G/Ho.

5.3. AFOCEL's network for Douglas-fir

The same model was directly fitted on Douglas-fir data, to test the flexibility of the form for other species in regular stand (Fig. 8 and Tab. IV). Comparatively to previous models for Douglas-fir, this model limits explanatory variables to the which are easily available to manager, ensures robust behavior inside the calibration area and improves total explained variance

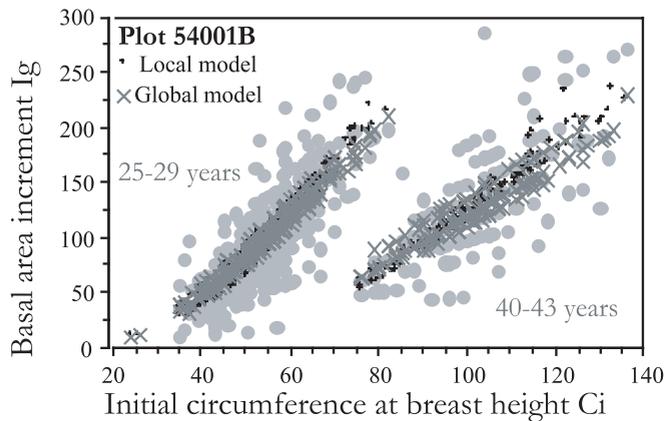


Figure 7. Basal area increment I_g (cm^2) vs. initial circumference at breast height (C_i , cm). Fitting example of final model, on Norway spruce trial 54001B with 2 growth periods (25–29 years and 40–43 years). By way of comparison, predictions of final model (black +) are given with those of initial local model (adjusted by stand, period and trial: grey x).

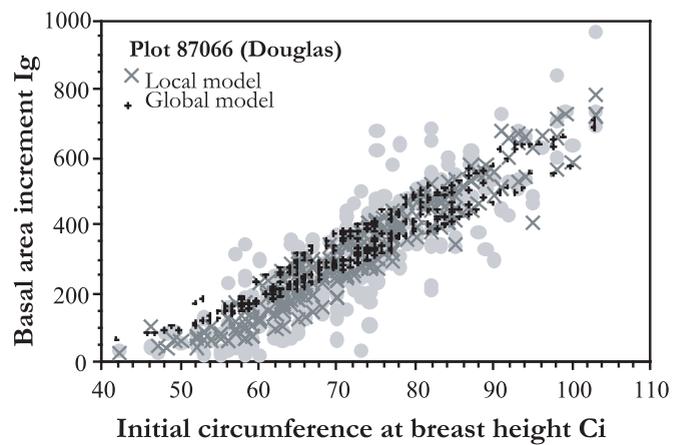


Figure 8. Basal area increment I_g (cm^2) vs. initial circumference at breast height (C_i , cm). Fitting example of final model, on Douglas-fir trial 87066 between ages 24 and 33. Predictions of final global model (black +) are given with those of initial local model (adjusted by stand, year and trial: grey x), by way of comparison.

(87.64% with a residual error: 1.1348 C_i cm). Generic capability of this growth model is then shown for pure even-aged stand for two main species.

A form parameter is slightly greater than for Norway spruce (1.03 instead 1.02) taking into account the more gradual

increase of increment between suppressed and dominant trees. This form means higher shade tolerance for Douglas-fir than Norway spruce for transitional trees.

These two models were finally implemented in the AFOCEL's simulation software OASIS (Fig. 9).

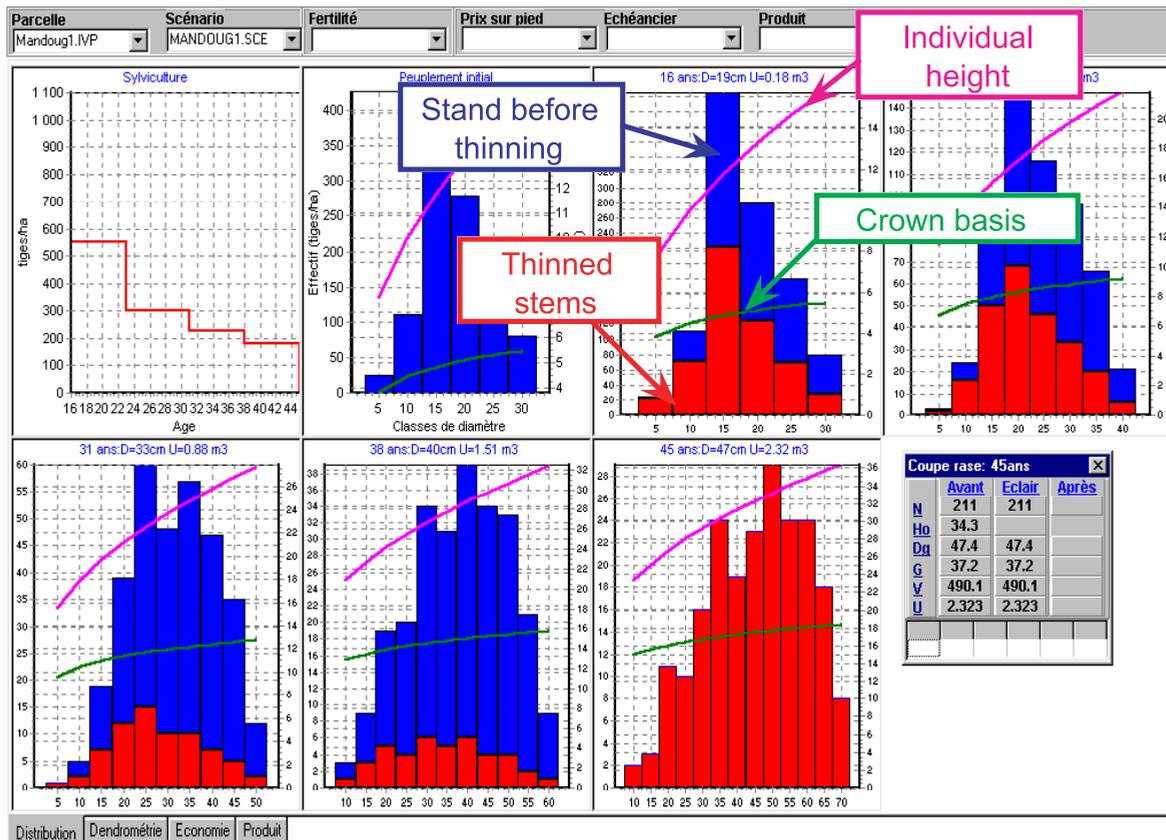


Figure 9. Simulation example of Douglas-fir model with OASIS with a dynamic silvicultural treatment.

6. DISCUSSION

The model of individual growth at breast height for Norway spruce, adapted to Douglas-fir, is the beginning of a generic growth model for regular (pure and even-aged) stands. Its constrained form partly retains the shape of Dhôte's model, which had proven to be effective to describe increments in different species (oak, beech, and spruce). But by generalizing the segmented Dhôte's model with a form parameter, a softer transition from suppressed trees towards dominant trees is allowed and it is slightly different according to species, as shown in this paper for Douglas-fir. A more general use of this model for various traditional species growing in regular stands could lead to a generic model, with only one form parameter adapted to each species.

Only stand information available to forest manager is required to implement the model. Available measurements needed are total basal area, dominant height and their increments.

The effect of period length between measurements on growth prediction has been particularly studied. Indeed if data is related to periods between 3 and 7 years, facility in using simulation software can quickly lead to annual reiterations of the model, or on the contrary to very large increment simulations (more than 10 years). As already studied by Dhôte [8], period length does not have a strong effect, except outside the data framework, i.e. for increments less than 3 years or more than 7 years.

Model use for other different species finally made it possible to start discussion on tree competition according to species characteristics. Using flexible, general models makes it possible to save energy by avoiding reformulating models and by allowing direct comparisons to be made [4]. To go further in this methodological step, the generic model should be tested on other major species. Such a tool would increase the coherence and robustness all growth models, and would allow a comparison between species, like different social behaviours, and would facilitate software implementation in simulators like OASIS with numerous species. A comprehensive description of different species in regular stands could be the first stage towards modelling mixed stands, which is a real challenge to modellers in the near future. Indeed operational tools for managers to simply describe mixed stands are just beginning.

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